

California Polytechnic State University

Economics 404  
International Trade Theory

Winter 2009  
Mr. Fisher

Midterm Examination

My name is Clever Mustang.

This exam is open everything.

1. Consider the following exchange economy. The set of agents is:

{Fisher, Morton, White}.

The endowments are:

$$\omega^F = \begin{bmatrix} 20 \\ 0 \end{bmatrix}, \omega^M = \begin{bmatrix} 0 \\ 10 \end{bmatrix}, \text{ and } \omega^W = \begin{bmatrix} 0 \\ 10 \end{bmatrix}.$$

All the agents have the same preferences summarized by the utility function

$$u(x_1, x_2) = \sqrt{x_1} \sqrt{x_2}.$$

- A. (2 points) Why is this a complete description of an exchange economy?  
**Because we have a list of agents, endowments, and preferences.**
- B. (5 points) Solve for the equilibrium, Show your work. **Let the price of the second good be one. Then the money income of Fisher is  $20 p_1$ , that of Morton is 10, and that of White is 10. Then the material balances equation for the first good is:  $10 \frac{p_1}{p_1} + \frac{5}{p_1} + \frac{5}{p_1} = 20$ . Hence  $p_1 = 1$ . The equilibrium allocations are**  
 **$x^F = \begin{bmatrix} 10 \\ 10 \end{bmatrix}, x^M = \begin{bmatrix} 5 \\ 5 \end{bmatrix}, \text{ and } x^W = \begin{bmatrix} 5 \\ 5 \end{bmatrix}$ .**
- C. (3 points) Explain why free trade Pareto dominates autarky. **Under autarky, each agent's utility is zero. Under free trade, each agent's utility is positive. So everyone prefers free trade to autarky.**

2. Consider the following table of labor coefficients.

| Hours to Make One Unit |      |      |        |
|------------------------|------|------|--------|
|                        | Wine | Beer | Cheese |
| England                | 1    | 2    | 3      |
| France                 | 10   | 20   | 29     |

- A. (2 points) Explain which country has absolute advantage in the first good.  
**England has absolute advantage in the first good because it takes less time to make it.**
- B. (2 points) Write down the chain of comparative advantage.  
**I will write it down with England's strongest comparative advantage on the left:  $1/10 = 1/10 < 3/29$**
- C. (6 points) England has 3000 person hours of labor and France has 29 person hours of labor. Each country has identical preferences represented by the utility function  $u(W, C, B) = W^{1/3}C^{1/3}B^{1/3}$ . Solve for the equilibrium. Show your work. **I guess that England produces all three goods since it is the large country. I will let the French wage be \$1. Then the English wage is \$29/3. French GDP is \$29, and English GDP is \$29,000. The price of the first good is \$29/3, the price of the second good is \$58/3, and the price of the third good is \$29 and it can be made in both France and England. World demand for the third good is  $(1/3)$29,029/29=1001/3$ . Hence some of it is produced in England since France can only make one unit of it. So my guess was correct.**

**The equilibrium allocations have England buying 1000 units of the first good, 500 units of the second good, and 1000/3 units of the third good. France buys one unit of the first good, 1/2 unit of the second good, and 1/3 unit of the third good.**

3. Here is a technology matrix. The first column is the unit input requirement for capital, and the second one is that for labor. There are three goods.

$$A = \begin{bmatrix} 1 & 1 \\ 2 & 1 \\ 3 & 1 \end{bmatrix}$$

The prices of the three goods are

$$p = \begin{bmatrix} 3 \\ 5 \\ 7 \end{bmatrix}$$

- A. (2 points) Which good is labor-intensive? Explain your answer. **The first sector is labor intensive because its capital labor ratio is only one. The other two sectors have higher capital-labor ratios.**
- B. (2 points) What is the cost share of labor in each industry? **I see that the rentals rate is 2 and the wage rate is 1. Then the cost share of labor in the first in industry is 1/3, in the second industry it is 1/5, and in the third industry it is 1/7.**
- C. (6 points) The price of the second good rises by 20%. What happens to the wage rate and the rentals rate? (Hint: there are three cases: (1) the first sector shuts down; (2) the third sector shuts down; and (3) both the first and third sectors shut down. This problem does not have a unique answer, but I am looking to see how deeply you can think. )

**Certainly, the second sector will continue to operate. Now  $2r + w = 6$ . So either the wage rate rises, the rentals rate rises, or both.**

**If the only wage rate rises, then the first sector shuts down. Hence  $3r + w = 7$  and  $2r + w = 6$ , and  $r = 1$  and  $w = 4$ . In this case, the costs in the first sector are 5, and they exceed its revenues**

**If only the rentals rate rises, then the third sector shuts down. Hence,  $r + w = 3$  and  $2r + w = 6$ . So  $r = 3$  and  $w = 0$ . In this case, the costs in the third sector are 9, and they exceed its revenues.**

**If both rise, then only the second sector can continue to operate. So if the wage is greater than 1, the rentals rate is greater than 2, and they add to 6, only the second sector will operate. Hence for  $4 > w > 1$ , and  $r = 6 - w$ , the first and the third sectors will shut down.**

4. Consider the same technology matrix

$$A = \begin{bmatrix} 1 & 1 \\ 2 & 1 \\ 3 & 1 \end{bmatrix}.$$

The economy has this endowment

$$\begin{bmatrix} K \\ L \end{bmatrix} = \begin{bmatrix} 600 \\ 300 \end{bmatrix}.$$

Prices are again

$$p = \begin{bmatrix} 3 \\ 5 \\ 7 \end{bmatrix}.$$

- A. (2 points) What is GDP? Show your work. You can use your answer from 3.B. **Since the rentals rate is 2 and the wage rate is 1, GDP = 1500**
- B. (4 points) Give two possible outputs that this economy can produce using all of its capital and labor. Show that both choices give the same GDP. Show your work. **One possibility is 100 of each of the three goods. This gives GDP = 300 + 500 + 700 = 1500. Another is to produce 100.5 of the first good, 99 of the second, and 100.5 of the third. Now I have GDP = 301.5 + 495 + 703.5 = 1500.**
- C. (4 points) The transpose of the pseudo-inverse of the technology matrix is:

$$A^{+T} = \begin{bmatrix} -1/2 & 4/3 \\ 0 & 1/3 \\ 1/2 & -2/3 \end{bmatrix}.$$

Does this make sense as a Rybczynski matrix? Explain your reasoning. **If the economy gets one more piece of capital, it can always decrease the output of the first sector by  $1/2$  and increase the output of the second sector by  $1/2$ . It does this by freeing up  $1/2$  unit of labor in the first sector to work with  $3/2$  unit of capital in the last sector. If the economy gets one more worker, it can always increase output in the first and second sectors by  $4/3$  and  $1/3$  respectively. It gets the extra resources by taking two pieces of capital and  $2/3$  of a worker away from the third sector.**

5. There are only two countries in the world. The technology matrix for America is  $A^{US} = \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}$  and the technology matrix for Mexico is  $A^{MEX} = \begin{bmatrix} 0.5 & 3 \\ 1 & 4 \end{bmatrix}$ . In these matrices, the first column represents inputs of capital and the second inputs of labor. World prices are  $p = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$ . America has this output vector  $y^{US} = \begin{bmatrix} 300 \\ 400 \end{bmatrix}$ , and Mexico has this output vector  $y^{MEX} = \begin{bmatrix} 800 \\ 100 \end{bmatrix}$ . America has these net exports  $x^{US} - m^{US} = \begin{bmatrix} -250 \\ 150 \end{bmatrix}$
- A. (2 points) What is Mexico's endowment? What is America's endowment?  
Mexico has 500 units of capital and 2800 units of labor. America has 1100 units of capital and 700 units of labor.
- B. (2 points) Explain why Mexico produces everything with a lower capital labor ratio than America. The capital labor ratio for the first good is  $1/6$ , and for the second good, it is  $1/4$ . The equivalent ratios in America are  $1/1$  and  $2/1$ .
- C. (2 points) Is there factor price equalization? Explain your answer. No. The US rentals rate is 2 and the US wage is 1, but the Mexican rentals rate is 3 and its wage rate is 0.5.
- D. (2 points) What is Mexico's GDP? What is America's GDP? Show your work. Mexico's GDP is  $2400 + 500 = 2900$ , and America's GDP is  $900 + 2000 = 2900$ .
- E. (2 points) Use the USA as a reference country. What is the measured factor content of US exports? It is  $\begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} -250 \\ 150 \end{bmatrix} = \begin{bmatrix} 50 \\ -100 \end{bmatrix}$ , where the top row is capital and the bottom row is labor.
- F. (4 points) What is the predicted factor content of US trade? Show your work. There are 1600 units of capital in the world. America is predicted to consume half of that. So it should export 300 units of capital. There are 3500 units of labor in the world. America is predicted to consume 1750 of it. So it should import 1050 units of it.
- G. (2 points) Use the USA as reference country. What is Mexico's virtual endowment? Show your work? It is  $\begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 800 \\ 100 \end{bmatrix} = \begin{bmatrix} 1000 \\ 900 \end{bmatrix}$ .
- H. (4 points) What is the predicted factor content of America's net exports using Mexico's virtual endowment? Show your work. Now there are 2100 units of capital in the world and 1600 units of labor. America should consume 1050 and 800 respectively. It has 1100 and 700. So it is predicted to export 50 units of capital and import 100 units of labor.