California Polytechnic State University

Economics 404 — Winter 2009
International Trade Theory — Mr. Fisher

Midterm Examination, **Answers**

My name is _ **Clever Mustang**_______________________________.

1. Consider a world that consists of two countries. Each country has identical preferences represented by this utility function:

$$u(x_1, x_2, x_3) = (1/4)\ln x_1 + (1/4)\ln x_2 + (1/2)\ln x_3$$

where x_i is consumption of good $i = 1,2,3$. The corresponding individual demand is given by

$$x(p_1, p_2, p_3; m) = \begin{bmatrix} m/4p_1 \\ m/4p_2 \\ m/2p_3 \end{bmatrix}$$

where p_i is the price of good $i = 1,2,3$ and m is money income. The endowments of the two countries are:

$$\omega^H = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \text{ and } \omega^H = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}.$$

A. (4 points) Explain why this is a complete description of an exchange economy. **It gives a list of agents, endowments, and preferences.**

B. (4 points) Show that the individual demands satisfy the homogeneity property and the adding up constraint.

$$x(2p_1, 2p_2, 2p_3; 2m) = \begin{bmatrix} 2m/8p_1 \\ 2m/8p_2 \\ 2m/4p_3 \end{bmatrix} = \begin{bmatrix} m/4p_1 \\ m/4p_2 \\ m/2p_3 \end{bmatrix} = x(p_1, p_2, p_3; m)$$

C. (4 points) Write down the condition describing equilibrium in the world economy. $x_H(p_1, p_2, p_3; m) + x_F(p_1, p_2, p_3; m) = \omega_H + \omega_F$ **In other words, supply must equal demand in every market.**

D. (4 points) Solve for the equilibrium. (Hint: it includes a list of prices and allocations.) $\begin{bmatrix} m_H/4p_1 \\ m_H/4p_2 \\ m_H/2p_3 \end{bmatrix} + \begin{bmatrix} m_F/4p_1 \\ m_F/4p_2 \\ m_F/2p_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$

$$\begin{bmatrix} (p_1+p_3)/4p_1 \\ (p_1+p_3)/4p_2 \\ (p_1+p_3)/2p_3 \end{bmatrix} + \begin{bmatrix} (p_2+p_3)/4p_1 \\ (p_2+p_3)/4p_2 \\ (p_2+p_3)/2p_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

Let $p_3 = 1$. Then

$$\begin{bmatrix} (p_1+1)/4p_1 \\ (p_1+1)/4p_2 \\ (p_1+1)/2 \end{bmatrix} + \begin{bmatrix} (p_2+1)/4p_1 \\ (p_2+1)/4p_2 \\ (p_2+1)/2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

These three equations are symmetric in the first two prices. Hence

$$p_1 = p_2$$

Then the third equation implies that

$$p_1 = p_2 = 1$$

Then the individual demands are

$$x_H(1,1,1;2) = \begin{bmatrix} 1/2 \\ 1/2 \\ 1 \end{bmatrix} \textbf{ and } x_F(1,1,1;2) = \begin{bmatrix} 1/2 \\ 1/2 \\ 1 \end{bmatrix}$$

E. (4 points) Describe the pattern of trade. **The home country exports ½ a unit of good one to the foreign country, and the foreign country exports ½ a unit of good 2 to the home country. There is no trade in good 3.**

2. Consider the following table of labor coefficients. Demand in each country is identical, and each country demands all three goods.

Hours to Make One Unit			
	Wine	Beer	Cheese
England	4 hours	5 hours	6 hours
France	1 hour	3 hours	2 hours

A. (5 points) For which goods does France have an absolute advantage? Explain your answer. **France has absolute advantage in all three goods because it takes fewer hours to make each of them than does England.**

B. (10 points) Write the chain of comparative advantage. **The chain of comparative advantage for England is 5/3 < 6/2 < 4/1. So England has strongest comparative advantage in beer and weakest comparative advantage in wine.**

C. (5 points) What, if anything, can one say about the pattern of trade? **England will never import beer, and France will never import wine.**

3. Consider a world consisting of three countries. There are two goods. The labor coefficients are given by

	Bread	Butter
Ameristan	1	3
Bobonia	2	2
Cacaland	3	1

Ameristan has 10 workers, Bobonia has 20 workers, and Cacaland has 10 workers. Each country always eats one piece of bread with one unit of butter; thus bread and butter are perfect complements.

A. (10 points) Draw the world production possibility frontier.

Put butter on the horizontal axis, and bread on the vertical axis. Then the world PPF has three linear segments. It starts at (18.33, 0) and goes up to (15, 10); this is where Ameristan starts to make bread . Then it goes through (10, 15); this is where Bobonia starts to make bread. Finally, it goes through (0, 18.33); this is where Cacaland starts to make bread.

B. (10 points) Solve for the pattern of trade.
The prices of butter and bread are both one. Ameristan makes 10 bread and exports 5 bread to

Cacaland, and imports 5 butter from them. Cacaland keep 5 butter for itself. Bobonia is in autarky, making 5 of each.

4. A country produces two goods. The first good is produced according to this production function: $Q_1 = \min\{K_1, L_1\}$, where the variables have their usual meanings. The second good has this production function: $Q_2 = \min\{K_2/2, L_2\}$. This country is endowed with 150 machines and 100 workers. All resources are fully employed.

   A. (5 points) Let the wage rate be $10 and the rentals rate $15. What is the marginal cost of each good? .

   The marginal cost of the first good is $MC_1 = w + r = \$25$ **and that of the second good is** $MC_2 = w + 2r = \$40$

   B. (5 points) Explain why the second good always uses two units of capital for each worker.

   The production function shows that capital and labor are perfect complements in production. In order to produce one unit of output, one needs at least one worker. Likewise, one need at least $K_2/2=1$ units of capital. Hence one needs two units of capital.

   C. (5 points) Explain why the first industry is labor-intensive, no matter what the wage-rentals ratio is.

   The first sector uses one unit of capital with each worker. The second sector uses two units of capital with each worker. It is obvious that the number of workers per unit of capital is higher in the first sector; thus is it labor-intensive.

   D. (5 points) Solve for outputs of both goods.

Using the full employment conditions for capital, we see that Q_1 = 50 and Q_2 = 50.

5. A country produces two goods. . The first good is produced according to this production function: $Q_1 = K_1^{1/2} L_1^{1/2}$, where the variables have their usual meanings. The second good has this production function: $Q_2 = K_2^{2/3} L_2^{1/3}$. The marginal product of capital in the first sector is $(1/2)K_1^{-1/2} L_1^{1/2}$

   A. (5 points) Explain why the rentals rate is the marginal value product of capital, no matter in which sector a machine is located.

A unit of capital is mobile between sectors. So it must earn the same return wherever it is located. The benefit of hiring a unit of capital is its marginal physical product. Thus it earns its marginal value product. Its cost is the rentals rate. Hence, the rentals rate equals the marginal value product of capital; otherwise the firm should rent more capital or get rid of capital, until the cost and benefit match.

   B. (5 points) Explain why the cost share of capital in the first sector is 50%.

The rentals rate is such that $rK_1 = P_1 K_1 (1/2) K_1^{-1/2} L_1^{1/2} = (1/2) P_1 Q_1$**.**
Hence payments for capital exhaust half of total revenues $P_1 Q_1$

   C. (10 points) The price of the first good rises by 1%, and the price of the second good is constant. What are the percentage changes in the wage rate and the rentals rate?

Solve two equations in two unknowns:

$$\hat{P}_1 = 1\% = (1/2)\hat{r} + (1/2)\hat{w}$$

$$\hat{P}_2 = 0 = (2/3)\hat{r} + (1/3)\hat{w}$$

$$\hat{w} = 4\%$$

$$\hat{r} = -2\%$$