Second Handout
Economics 404, Cal Poly

1. Here are some reminders about matrix multiplication.

   (a) A matrix is has r rows and c columns. Here are some examples

   $$\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ -2 & 5 \end{bmatrix}$$

   $$\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$$

   $$\begin{bmatrix} a & b & c & d \\ e & f & g & h \end{bmatrix}$$

   The first matrix has 3 rows and 2 columns, the second one has 3 rows and 1 colum, and the last one has 2 rows and 4 columns. A matrix with only one column is called a vector. A matrix with only one row is called a row vector. A matrix with one row and one colum is called a scalar..

   (b) Transposing a matrix flips its columns and rows. Here are some examples

   $$\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ -2 & 5 \end{bmatrix}^T = \begin{bmatrix} 1 & 3 & -2 \\ 2 & 4 & 5 \end{bmatrix}$$

   $$\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}^T = \begin{bmatrix} 1 & 2 & 0 \end{bmatrix}$$

   $$\begin{bmatrix} a & b & c & d \\ e & f & g & h \end{bmatrix}^T = \begin{bmatrix} a & e \\ b & f \\ c & g \\ d & h \end{bmatrix}$$

(c) Matrix multiplication is accomplished by forming the inner products of a vector with the rows of a conformable matrrix. Here are some examples

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ -2 & 5 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 13 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} 3 = \begin{bmatrix} 3 \\ 6 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} a & b & c & d \\ e & f & g & h \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} = \begin{bmatrix} a + 2b + 3c + 4d \\ e + 2f + 3g + 4h \end{bmatrix}$$

2. Here's how to inverst a square matrix with two rows and two columns. Let

$$X = \begin{bmatrix} a & b \\ c & d \end{bmatrix}.$$

Then

$$X^{-1} = (1/(ad - bc)) \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}.$$

If $(ad - bc) = 0$, then the matrix does not have an inverse.

(a) You can use the inverse of a matrix to solve problems. If $Ax = b$, then

$$x = A^{-1}b$$

(b) Here are two examples

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix} \Longrightarrow \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = (-1/2) \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 0 \end{bmatrix} = \begin{bmatrix} -4 \\ 3 \end{bmatrix}$$

and

$$\begin{bmatrix} 1 & 2 \\ 0 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \Longrightarrow \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = (1/5) \begin{bmatrix} 5 & -2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1/5 \\ 2/5 \end{bmatrix}.$$