Chance and Irrationality in Demand Theory

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Abstract

This paper examines and characterizes the economic behavior of individuals whose choices are not based on a preference order and maximizing behavior. The paper shows that average demand curves for individuals whose decisions are left to random chance are homogeneous in prices and income, add up to total income, satisfy the Slustky equation, and have a negative semi-definite matrix of substitution effects. These properties hold for any well-behaved distribution function and, excluding adding up, for allocations that are in the interior of the budget set. The paper also discusses the implications of irrational behavior for the assumption of rationality, the role of preferences in economic behavior, aggregation and individual irrationality, and irrationality and welfare.

Keywords: Economic behavior; demand theory; random choice; irrationality

JEL classification: D01; D11; C02

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1 Introduction

Virtually all discussions in economics assume the existence of well-defined preferences and maximizing behavior. Until recent decades, however, economists have paid relatively little attention to the origin and nature of preferences. Economic analyses often take preferences as given and assume individuals make rational choices to maximize personal satisfaction by obtaining goods and services in the market.

This absence of a clear understanding of preferences has prompted a series of important behavioral analyses that seek to give a realistic foundation to economic choices. Recognizing that preferences and rationality are critical in most economic models, the purpose of this paper is not to explore the biological, cultural, evolutionary, social, or psychological foundations of preferences. In fact, this paper disposes of preferences and the maximizing behavior that is typical of rational models. This paper takes further steps towards better understanding the foundations of economic behavior by studying a model of irrational behavior, a generalization of the model proposed by Gary Becker (1962).

This paper focuses on individuals whose decisions are *irrational*.[1] There are two goods in the economy, X and Y, and the individuals' income $\mathbf{I}$ must be divided so that the budget constraint is satisfied:

$$\mathbf{P}_x X + \mathbf{P}_y Y \leq \mathbf{I}, \tag{1}$$

with $X \geq 0$, and $Y \geq 0$, and with $\mathbf{P}_x$ and $\mathbf{P}_y$ as the given prices of X and Y.

[1] Irrationality here, and in Becker (1962), represents the absence of preference orderings and maximizing behavior. Other notions of irrationality can be seen as based on small departures from optimizing behavior (as in Akerlof and Yellen, 1985), as choices based on limited or bounded rationality (as in Simon, 1979), as habitual choices (as discussed in Arrow, 1986) or as imitative behavior (as in Conlisk, 1980). A strict definition of irrationality is not attempted here because individuals behave irrationally in many ways.

Typically, the demand functions for X and Y assume that individuals have a well-defined preference order or a utility function $U(X,Y)$ which is maximized subject to the budget constraint (1). Here, in contrast, individuals behave irrationally in the following sense: taking prices and income as given, individuals let random chance or luck decide their consumption of X subject to $\mathbf{P}_x X \leq \mathbf{I}$. Once X is obtained, individuals either spend the rest of their income in good Y or they let random chance also decide their demand for good Y. If random chance also determines Y, the joint choice of X and Y must be affordable or satisfy (1).

What kind of behavior can we expect from such irrational individuals? Surprisingly, Becker (1962) showed that we can expect irrational individuals to behave much in the same way as rational individuals behave in one central dimension. In response to an increase in the price of X, rational and irrational individuals alike will, *ceteris paribus*, demand lower amounts of good X or satisfy the 'Law of Demand.'

Becker's (1962) seminal model relies on two very special assumptions that will be relaxed here. First, all possible values of X have equal chance of being randomly selected. Further, individuals spend all of their income in X and Y. Since X is uniformly distributed, the average demand for good X in Becker (1962) is:

$$\bar{x}(\mathbf{P}_x, \mathbf{P}_y, \mathbf{I}) := \mathbb{E}\left[X \,|\, 0 \leq X \leq \mathbf{I}/\mathbf{P}_x\right] = \frac{1}{\mathbf{I}/\mathbf{P}_x} \int_0^{\mathbf{I}/\mathbf{P}_x} x dx = \frac{1}{2}\frac{\mathbf{I}}{\mathbf{P}_x}, \tag{2}$$

while the average demand for good Y is : $\bar{y}(\mathbf{P}_x, \mathbf{P}_y, \mathbf{I})\mathbf{P}_y = \frac{1}{2}\frac{\mathbf{I}}{\mathbf{P}_y}$.

These demands are clearly well-behaved. A collection of irrational selves will behave *on average* in the same manner as rational individuals endowed with a utility function

$U(X, Y) = XY$ or any monotone transformation of it.[2] The assumption of a uniform distribution, however, unnecessarily restricts Becker's (1962) model. Becker (1962) also emphasized the 'Law of Demand' and choices in the boundary of the budget set.

This paper considers random choices for general probability distribution functions. This paper also allows choices in the interior of the budget set. Further, this paper fully characterizes all of the properties of preference-based analyses for the demand functions associated with random choice. Generalizations that consider many goods, many agent-types, and some general equilibrium aspects are also a developed here. None of these aspects has been a subject of study in the previous literature.[3]

There are several important insights to be gained from a general model of irrational choice. First, this paper shows that the 'Law of Demand' holds for *any* well-behaved distribution function and not just for the uniform distribution. Second, the 'Law of Demand' holds even if choices are *inside* the budget set (1). In both cases, demand functions satisfy the Slustky equation. Finally, this paper demonstrates that demand functions for irrational individuals satisfy *all* of the properties of the preference-based demand theory: average demands are homogeneous of degree zero in prices and income, have symmetric cross-price effects, a negative semi-definite matrix of substitution effects, and satisfy

[2] Because 'individual' demands are not measurable, the paper considers throughout average demands. Other measures of central tendency are studied later on. The findings in this paper are also important for the integrability problem or the possible recovery of preference relations from observed choices. (See Mas-Colell et al. (1995, Section 3H) for a general presentation of this problem.) Average demands in this paper are integrable although, as noted by Blundell et al. (2003, 211), "it is not clear that we would wish to characterize [irrational demand functions] as the outcome of a 'rational' procedure."

[3] There are only a few generalizations of Becker (1962). Sanderson (1974) is an important study that relates to this paper although the discussion is very different from the approach taken here. In Sanderson (1974), the distribution functions need to be restricted for irrational demands to be well-behaved. Here, however, no restriction is needed. The analysis automatically satisfies the orderings assumed in Sanderson (1974). Sanderson (1974, 1980) also extended the model of irrational choice to household production settings. This paper does not pursue that possibility.

adding up conditions.

By showing that a model of irrational choice gives general results that are observationally equivalent to models of rational choice, this paper strengthens Becker's (1962) claim that a rigorous derivation of demand curves can be achieved without relying on preference relationships and individual rationality.[4] Contrary to the common belief, utility functions and the assumption of maximizing behavior are not always essential for deriving the most basic economic principles in demand theory.[5] (See also Deaton and Muellbauer (1980, Chapter 1) and Becker (1971, Lesson 7) for related discussions.)

Showing that any distribution function generates average demand functions that resemble rational choices not only verifies that aggregation is responsible for rationality in aggregate outcomes but also proves that no general restriction on the shape of the distribution of heterogeneity is needed for aggregation to work. This is important because the uniform distribution in Becker (1962) has a flat density function and it is symmetric. Symmetry suggests that some balancing effect might be responsible for the aggregation effect while the flatness of the uniform density suggests that the nice properties of aggregation rely on some form of aggregate insensitiveness, see, e.g., Grandmont (1992).

Proving that the 'Law of Demand' holds even if irrational choices are allowed to be

[4] Individuals in Becker (1962) are "impulsive good deciders" as they let chance decide the amount of goods consumed. "Impulsive money deciders" let chance decide the amount of money spent in each good, see Chant (1963). A distinction between these type of choices has been exploited in experimental settings for rats, pigeons and other animals by Battalio et al. (1981a, 1981b, 1987). Most of the comparisons though use the uniform distribution. This is also true in Bronars (1987) and in Cox (1997) where irrational behavior is used to test the hypothesis of utility maximization.

[5] Symmetry in the matrix of substitution effects is commonly associated with consistent preferences or seen as a direct consequence of Axioms of rational choice, see Deaton and Muellbauer (1980, 45) and Mas-Colell et al. (1995, Chapter 2). In rational choice, a symmetric negative semi-definite matrix of substitution effects is a consequence of concavity in the utility function and of the symmetry of second derivatives or Young's Theorem. It is important to stress that Young's Theorem, the Axioms of revealed preference, or the Axioms of rational choice are of no use here.

inside the budget set is also important because in Becker (1962), decisions are always non-interior. In an abuse of notation, individuals in Becker (1962) prefer more to less as their random choices (almost) always exhaust income. The assumption that "more is preferred to less" is central in preference analyses. Non-satiation is responsible for downward sloping indifference curves and for the direction of increasing utility in the indifference map. Moreover, non-satiation ensures homogeneity, adding up conditions, and it is an assumption that eliminates 'money illusion.'

In the interest of brevity, the next Theorem summarizes the findings of the paper:

Theorem 1 (Irrational behavior) *Assume individuals let random chance, as defined above, decide X and Y subject to (1). Then, for every $(\mathbf{P}_x, \mathbf{P}_y, \mathbf{I})$, average demand functions $\bar{x}(\mathbf{P}_x, \mathbf{P}_y, \mathbf{I})$ and $\bar{y}(\mathbf{P}_x, \mathbf{P}_y, \mathbf{I})$:*

- *Satisfy the 'Law of Demand' and the Slustky decomposition,*
- *Are homogeneous of degree zero in prices and income,*
- *Have a symmetric and negative semi-definite matrix of substitution effects, and*
- *Add up to income.*

The previous results hold for any (well-behaved) distribution of X and Y, and, excluding adding up, for allocations in the interior of the budget set (1).

The rest of this paper is devoted to the proof of the main result, to generalizations, and to the interpretation of Theorem 1. The analysis of non-interior allocations is in Section 2. Section 3 examines interior allocations. Section 4 considers additional forms of random behavior, the case of log-concavity in the distribution of X, and presents a simulation exercise. Further, Section 4 considers an economy with many goods, many individual-types, and a general equilibrium elaboration in a pure exchange economy. The interpretation of Theorem 1 and the importance of irrational behavior are discussed in the concluding remarks in Section 5. This paper offers remarks on four fundamental aspects: the assumption of rationality, the role of preferences in economic behavior, aggregation and individual irrationality, and irrationality and welfare.

2 Non-interior allocations

This section of the paper considers a case in which only X is randomly selected. Once X is determined, the remaining income is spent in good Y. As all income is eventually spent, the demands lie in the boundary of the budget set. Throughout this sub-section, we assume that the random variable X has a probability density function $f(x)$ and a cumulative distribution function $F(x)$ where $f(x) := \frac{dF(x)}{dx}$ is continuous, differentiable, and positive for $x \in [0, +\infty)$. (Throughout the paper, we assume that income and prices are strictly positive and finite.)

The conditional expectation that defines the average demand for X is given by a well-known *right-truncated mean* formula:

$$\bar{x}(\mathbf{P}_x, \mathbf{P}_y, \mathbf{I}) := \mathbb{E}\left[X \,|\, 0 \leq X \leq \mathbf{I}/\mathbf{P}_x\right] = \frac{\int_{\mathbf{0}}^{\mathbf{I}/\mathbf{P}_x} x f(x) dx}{F(\mathbf{I}/\mathbf{P}_x)}, \tag{3}$$

with $0 \leq X \leq \mathbf{I}/\mathbf{P}_x$ and $F(\mathbf{I}/\mathbf{P}_x) := \Pr[0 \leq X < \mathbf{I}/\mathbf{P}_x] > 0$. Decisions, in the absence of the budget constraint (1), would be based on $f(x)$. Since not all choices are feasible, the relevant density function is a *conditional* density $f(x)/F(\mathbf{I}/\mathbf{P}_x)$. Assuming that choices are made in such a way implies that changes in the budget constraint affect the support of the conditional density function without changing the support or the shape of $f(x)$.

Because all remaining income is spent in Y once X is determined, the demand for Y is given by $\mathbf{P}_y Y = \mathbf{I} - \mathbf{P}_x X$ as long as the events considered are feasible. The events that defined the demand for X were those given by $0 \leq \mathbf{P}_x X \leq \mathbf{I}$. The events that define the demand for Y are $\mathbf{I} \geq \mathbf{P}_y Y \geq 0$. The case of Y is thus analogous to the demand for X since in both cases decisions can be seen as taking place in only one variable. The average demand for Y can be written as:

$$\bar{y}(\mathbf{P}_x, \mathbf{P}_y, \mathbf{I}) := \frac{\mathbf{I} - \bar{x}(\mathbf{P}_x, \mathbf{P}_y, \mathbf{I})\mathbf{P}_x}{\mathbf{P}_y}. \tag{4}$$

Next consider *price effects*. As usual, a change in $\mathbf{P}_x$ produces two conceptually separate effects on the opportunity set: First, the relative scarcity of good X changes with changes in the price (a substitution effect), and second, the real value of income available for purchases changes (an income effect). It is then necessary to compensate individuals in order to derive a meaningful 'Law of Demand.' The significance of the Slustky or Marshallian *equivalent compensation* that follows is well-known. When prices $\mathbf{P}_x$ change, the value $\bar{x}(\mathbf{P}_x, \mathbf{P}_y, \mathbf{I})\mathbf{P}_x$ is reduced (or increased) so in order to make possible the purchase of the same quantities of $\bar{x}(\mathbf{P}_x, \mathbf{P}_y, \mathbf{I})$ income should have to vary according to:

$$\left.\frac{\partial \mathbf{I}}{\partial \mathbf{P}_x}\right|_{\bar{\mathbf{I}}} = \bar{x}(\mathbf{P}_x, \mathbf{P}_y, \mathbf{I}), \tag{5}$$

and similarly for changes in $\mathbf{P}_y$. (From now on, $\bar{\mathbf{I}}$ represents income-compensated changes.)

2.1 The 'Law of Demand'

The next Lemma studies the compensated response to changes in $\mathbf{P}_x$ or the 'Law of Demand:'

Lemma 1 (Law of Demand I) *Assume individuals let chance decide X subject to $0 \leq X \leq \mathbf{I}/\mathbf{P}_x$. Then, for every $(\mathbf{P}_x, \mathbf{P}_y, \mathbf{I})$, $\bar{x}(\mathbf{P}_x, \mathbf{P}_y, \mathbf{I})$ and $\bar{y}(\mathbf{P}_x, \mathbf{P}_y, \mathbf{I})$ satisfy the 'Law of Demand,'*

$$\left.\frac{\partial \bar{x}(\mathbf{P}_x, \mathbf{P}_y, \mathbf{I})}{\partial \mathbf{P}_x}\right|_{\bar{\mathbf{I}}} \leq 0, \text{ and } \left.\frac{\partial \bar{y}(\mathbf{P}_x, \mathbf{P}_y, \mathbf{I})}{\partial \mathbf{P}_y}\right|_{\bar{\mathbf{I}}} \leq 0, \tag{6}$$

and have price effects associated with the Slustky equations:

$$\frac{\partial \bar{x}(\mathbf{P}_x, \mathbf{P}_y, \mathbf{I})}{\partial \mathbf{P}_x} = \left.\frac{\partial \bar{x}(\mathbf{P}_x, \mathbf{P}_y, \mathbf{I})}{\partial \mathbf{P}_x}\right|_{\bar{\mathbf{I}}} - \bar{x}(\mathbf{P}_x, \mathbf{P}_y, \mathbf{I}) \left[\frac{\partial \bar{x}(\mathbf{P}_x, \mathbf{P}_y, \mathbf{I})}{\partial \mathbf{I}}\right], \text{ and} \tag{7}$$

$$\frac{\partial \bar{y}(\mathbf{P}_x, \mathbf{P}_y, \mathbf{I})}{\partial \mathbf{P}_y} = \left.\frac{\partial \bar{y}(\mathbf{P}_x, \mathbf{P}_y, \mathbf{I})}{\partial \mathbf{P}_y}\right|_{\bar{\mathbf{I}}} - \bar{y}(\mathbf{P}_x, \mathbf{P}_y, \mathbf{I}) \left[\frac{\partial \bar{y}(\mathbf{P}_x, \mathbf{P}_y, \mathbf{I})}{\partial \mathbf{I}}\right]. \tag{8}$$

Proof. (First part) We begin the proof with a definition. Let $\mathbf{Z} := \mathbf{I}/\mathbf{P}_x$ represent the maximum quantity of X that can be purchased for a given income $\mathbf{I}$ and price $\mathbf{P}_x$. The limit of integration in (3) is given by $\mathbf{Z}$. The previous assumptions on $F(x)$ ensure continuity and differentiability in $\bar{x}(\mathbf{P}_x, \mathbf{P}_y, \mathbf{I})$. The relevant derivative in equation (3) is:

$$\left.\frac{\partial \bar{x}(\mathbf{P}_x, \mathbf{P}_y, \mathbf{I})}{\partial \mathbf{P}_x}\right|_{\bar{\mathbf{I}}} = \frac{\partial \mathbb{E}\left[X \,|\, 0 \leq X \leq \mathbf{Z}\right]}{\partial \mathbf{Z}} \left(\left.\frac{\partial \mathbf{Z}}{\partial \mathbf{P}_x}\right|_{\bar{\mathbf{I}}}\right), \tag{9}$$

in which the first term represents the effect of changes in $\mathbf{Z}$ on the average demand and the second the effect of compensated changes in prices on $\mathbf{Z}$. This first term satisfies:

$$\frac{\partial \mathbb{E}\left[X|\, 0 \leq X \leq \mathbf{Z}\right]}{\partial \mathbf{Z}} = \frac{f(\mathbf{Z})}{F(\mathbf{Z})}\left(\mathbf{Z} - \mathbb{E}\left[X|\, 0 \leq X \leq \mathbf{Z}\right]\right) \geq 0,$$

by a standard application of Leibnitz rule for differentiation under the integral sign. The second term can be obtained from the budget constraint (1) and equation (5). In particular, under the income compensation in equation (5), the upper limit of integration $\mathbf{Z}$ changes according to:[6]

$$\left.\frac{\partial \mathbf{Z}}{\partial \mathbf{P}_x}\right|_{\bar{\mathbf{I}}} = -\frac{\mathbf{I} - \bar{x}(\mathbf{P}_x, \mathbf{P}_y, \mathbf{I})\mathbf{P}_x}{\mathbf{P}_x^2} = -\left(\frac{\bar{y}(\mathbf{P}_x, \mathbf{P}_y, \mathbf{I})}{\mathbf{P}_x}\right)\left(\frac{\mathbf{P}_y}{\mathbf{P}_x}\right) \leq 0.$$

The previous expressions give the 'Law of Demand' as the compensated change in $\mathbf{Z}$ is always negative and $\partial \mathbb{E}\left[X|\, 0 \leq X \leq \mathbf{Z}\right] / \partial \mathbf{Z}$ is always positive (or at least non-negative). To obtain the Slustky equation notice that: $\left.\frac{\partial \mathbf{Z}}{\partial \mathbf{P}_x}\right|_{\bar{\mathbf{I}}} = \frac{\partial \mathbf{Z}}{\partial \mathbf{P}_x} + \frac{\bar{x}(\mathbf{P}_x, \mathbf{P}_y, \mathbf{I})}{\mathbf{P}_x}$. Notice also that:

$$\frac{\partial \bar{x}(\mathbf{P}_x, \mathbf{P}_y, \mathbf{I})}{\partial \mathbf{I}} = \frac{\partial \mathbb{E}\left[X|\, 0 \leq X \leq \mathbf{Z}\right]}{\partial \mathbf{Z}}\left(\frac{\partial \mathbf{Z}}{\partial \mathbf{I}}\right) = \frac{\partial \mathbb{E}\left[X|\, 0 \leq X \leq \mathbf{Z}\right]}{\partial \mathbf{Z}}\frac{1}{\mathbf{P}_x}.$$

Substitution of the previous expressions completes the first part of the proof. The rest of the proof is available in the Appendix to this paper. ■

[6] To study compensated changes it is important to note that a Hicksian compensation is clearly not possible as there is no utility function. Also, no duality results can be derived for this problem since the expenditure function and Hicksian demands rely on a utility function. It is well-known that a Marshallian compensation of income is equivalent to a Hicksian compensation for very small changes in prices. Compensations that leave purchasing power unchanged can also be made through prices but the results are equivalent to income-based compensations. For a discussion of these topics see Friedman (1962). Finally, notice that one can indeed derive a measure of *consumer surplus* in this economy but such a measure has no meaningful interpretation because this paper is not using preferences to examine choices.

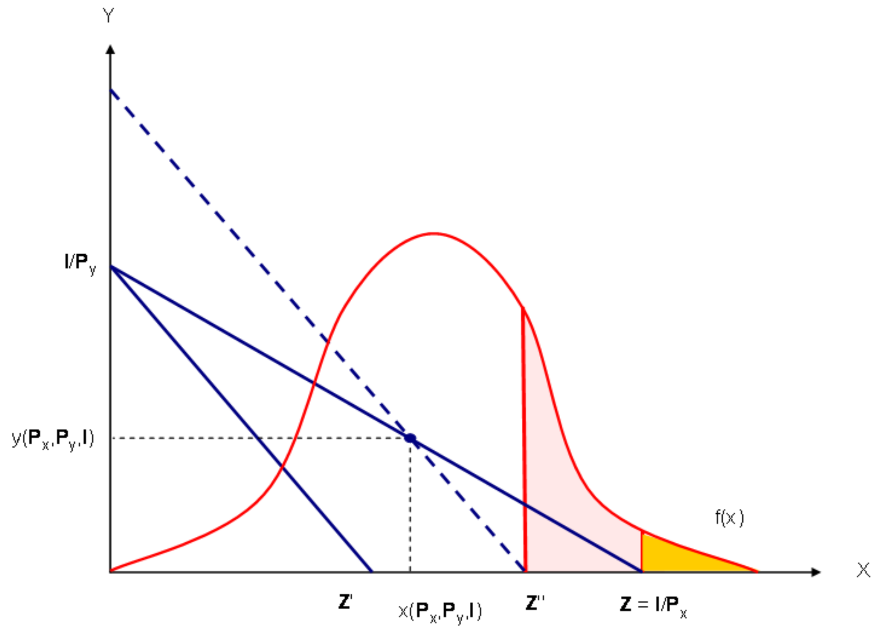

Figure 1: The average demand for good X, $\bar{x}(\mathbf{P}_x, \mathbf{P}_y, \mathbf{I})$, is drawn from $f(x)$. Feasible choices induce a truncated distribution with the truncation point given at $\mathbf{Z} = \mathbf{I}/\mathbf{P}_x$. As $\mathbf{P}_x$ increases, feasible choices are restricted further and average demand declines. This is true whether price changes are compensated (as in $\mathbf{Z}''$) or not (as in $\mathbf{Z}'$).

The previous Lemma is a general statement about the 'Law of Demand' that holds for any distribution function. The proof of Lemma 1 relies on one economic fact and one statistical fact. First, the maximum amount of good X that can be purchased declines as $\mathbf{P}_x$ increases. (This is true whether changes are compensated or not.) Second, the right-truncated mean increases as the point of truncation increases: $\partial \mathbb{E}\left[X \,|\, 0 \leq X \leq \mathbf{Z}\right] / \partial \mathbf{Z} \geq 0$. That is, the truncated mean is (almost) always less than the truncation point.[7]

[7] Measures of central tendency other than the mean and the mid-range fail to give a general 'Law of Demand.' The median is defined as: $\bar{x}_\mu(\mathbf{P}_x, \mathbf{P}_y, \mathbf{I}) := \{0 \leq x \leq \mathbf{Z} : F(x|0 \leq X \leq \mathbf{Z}) = 1/2\}$, and the mode (assuming differentiability and unimodality) as $\bar{x}_m(\mathbf{P}_x, \mathbf{P}_y, \mathbf{I}) := \{0 \leq x \leq \mathbf{Z} : \partial f(x|0 \leq X \leq \mathbf{Z})/\partial x = 0\}$. As before, one can write the median and mode as a function of $\mathbf{Z}$. The following results can be established from basic differentiation: $\partial \bar{x}_\mu(\mathbf{Z})/\partial \mathbf{Z} = \left[f(x_\mu) - f(\mathbf{Z})/2\right]/F(\mathbf{Z})$, and $\partial \bar{x}_m(\mathbf{Z})/\partial \mathbf{Z} = 0$, as long as the mode is interior. In neither case it is possible to derive general results about price changes. For the median, the response depends on the symmetry of the distribution while the mode will only change if the mode is an extreme point. Both points can be seen graphically in Figure 1.

The proof of Lemma 1 is described graphically in Figure 1. The horizontal axis gives the support of $F(x)$. The fact that some choices are not feasible truncates the distribution of X at $\mathbf{Z}$ and induces a relevant distribution function $F(x|0 \leq X \leq \mathbf{Z}) = F(x)/F(\mathbf{Z})$.[8] Once prices change, the truncation point changes to $\mathbf{Z}' < \mathbf{Z}$ when no compensation is given and to $\mathbf{Z}'' < \mathbf{Z}$ when income is compensated. In either case, the maximum quantity of X that can be purchased declines and the feasible choices of X are restricted even more. This forces the average value of X to decline from a pure *scarcity* principle. Similarly, an increase in $\mathbf{P}_x$ makes Y more "attractive" and, hence, its average demand increases.

Lemma 1 shows that the 'Law of Demand' can be established for any probability distribution and that the total price effect will be just as in the standard Slustky equation. This Slustky decomposition, as in the case of rational choices, has two terms. The first term represents a 'substitution effect' associated with a change in favor of the least expensive good. This substitution effect is purely mechanic as noted originally by Becker (1962). There are no intrinsic motivations behind it other than the relative scarcity brought by an increase in the price $\mathbf{P}_x$. The second term in (7) is related to how the purchasing power of individuals changes as prices change. As in the case of rational choices, it represents an 'income effect' derived entirely from changes in the opportunity set. This income effect is always positive.[9]

[8] In Sanderson (1974), the distribution function is restricted into producing a shift as the one here. No restriction is needed in this paper since the relevant distribution is the conditional distribution, $F(x|0 \leq X \leq \mathbf{Z})$. Being a conditional distribution, the stochastic dominance orderings assumed by Sanderson (1974) are obtained here. For example, $F(x|0 \leq X \leq \mathbf{Z}')$ stochastically dominates $F(x|0 \leq X \leq \mathbf{Z})$ in the first degree sense in $[0, \mathbf{Z}']$. Therefore, when the truncation point declines, the truncated mean also declines.

[9] The interpretation in Becker (1962, 8), and in most of the literature, is one in which "households may be irrational and yet markets are quite rational." Aggregation would still work even if decisions are not channeled through actual markets; i.e., if $\mathbf{P}_x$ and $\mathbf{P}_y$ represent *shadow prices*. Further, to see that heterogeneity is essential, consider a Dirac delta measure. The Dirac delta distribution function is defined by $F(\{x_\delta\}) = 1$ with measure one for the singleton $\{x_\delta\}$. The mean under this distribution is x_δ and the

2.2 Additional properties

Next, consider additional properties characteristic of demand functions. The properties we consider are: homogeneity, symmetry, and adding up. Homogeneity describes the response of average demands to a proportional change in all prices and in income. Symmetry refers to the response to cross-price changes, and adding up to the relationship between the value of demands and total income. (These properties are the only ones associated with classical demand theory, see Mas-Colell et al. (1995, Chapter 2).)

As the next Lemma shows, irrational demands are homogeneous of degree zero in prices and income (or there is no 'money illusion'), symmetric, and they (almost) always satisfy adding up conditions:

Lemma 2 (Homogeneity, symmetry, and adding up) *For every* $(\mathbf{P}_x, \mathbf{P}_y, \mathbf{I})$*, under the conditions of Lemma 1,* $\bar{x}(\mathbf{P}_x, \mathbf{P}_y, \mathbf{I})$ *and* $\bar{y}(\mathbf{P}_x, \mathbf{P}_y, \mathbf{I})$ *are homogeneous of degree zero in prices and income, the cross-price derivatives are symmetric, and demands satisfy the adding up condition.*

Proof. Homogeneity in $\bar{x}(\mathbf{P}_x, \mathbf{P}_y, \mathbf{I})$ follows by definition as only $\mathbf{Z}$ determines the demand for X, see equation (3). The demand for Y is also homogeneous as it follows from equation (4). Adding up conditions are satisfied by construction.

To prove that average demands are symmetric, consider first cross-price effects in the demand for X. Following steps similar to those in the proof of Lemma 1:

$$\left.\frac{\partial \bar{x}(\mathbf{P}_x, \mathbf{P}_y, \mathbf{I})}{\partial \mathbf{P}_y}\right|_{\bar{\mathbf{I}}} = \frac{\partial \mathbb{E}\left[X \middle| 0 \leq X \leq \mathbf{Z}\right]}{\partial \mathbf{Z}} \left(\frac{\bar{y}(\mathbf{P}_x, \mathbf{P}_y, \mathbf{I})}{\mathbf{P}_x}\right).$$

variance is zero. Any compensated price change will leave demands unchanged (almost everywhere).

Consider next the effect of compensated changes in $\mathbf{P}_x$ on the demand for good Y:

$$\left.\frac{\partial \bar{y}(\mathbf{P}_x, \mathbf{P}_y, \mathbf{I})}{\partial \mathbf{P}_x}\right|_{\bar{\mathbf{I}}} = \left(\left.\frac{\partial(\mathbf{I}/\mathbf{P}_y)}{\partial \mathbf{P}_x}\right|_{\bar{\mathbf{I}}}\right) - \left.\frac{\partial \bar{x}(\mathbf{P}_x, \mathbf{P}_y, \mathbf{I})}{\partial \mathbf{P}_x}\right|_{\bar{\mathbf{I}}} \frac{\mathbf{P}_x}{\mathbf{P}_y} - \frac{\bar{x}(\mathbf{P}_x, \mathbf{P}_y, \mathbf{I})}{\mathbf{P}_y}.$$

This expression follows from equation (4). Further, $\left(\left.\frac{\partial(\mathbf{I}/\mathbf{P}_y)}{\partial \mathbf{P}_x}\right|_{\bar{\mathbf{I}}}\right) = \frac{\bar{x}(\mathbf{P}_x, \mathbf{P}_y, \mathbf{I})}{\mathbf{P}_y}$ from the budget constraint (1) and equation (5). Thus, the previous expression can be written as:

$$\left.\frac{\partial \bar{y}(\mathbf{P}_x, \mathbf{P}_y, \mathbf{I})}{\partial \mathbf{P}_x}\right|_{\bar{\mathbf{I}}} = -\left.\frac{\partial \bar{x}(\mathbf{P}_x, \mathbf{P}_y, \mathbf{I})}{\partial \mathbf{P}_x}\right|_{\bar{\mathbf{I}}} \frac{\mathbf{P}_x}{\mathbf{P}_y} = \left(\frac{\partial \mathbb{E}\left[X \,|\, 0 \leq X \leq \mathbf{Z}\right]}{\partial \mathbf{Z}}\right)\left(\frac{\bar{y}(\mathbf{P}_x, \mathbf{P}_y, \mathbf{I})}{\mathbf{P}_x}\right),$$

where the last equality uses equation (9). ■

The next Lemma shows that average demands are also associated with a negative definite matrix of substitution effects:

Lemma 3 (Slustky matrix) *For every* $(\mathbf{P}_x, \mathbf{P}_y, \mathbf{I})$*, under the conditions of Lemma 1, the Slustky matrix of substitution effects,* $\mathcal{S}(\mathbf{P}_x, \mathbf{P}_y, \mathbf{I})$*, is symmetric and negative definite, and* $|\mathcal{S}(\mathbf{P}_x, \mathbf{P}_y, \mathbf{I})| = 0$*. Thus, average demand functions are integrable, i.e., demands can be 'rationalized.'*

Proof. Recall that the matrix of substitution effects is negative semi-definite if the 'Law of Demand' is satisfied and if the determinant of the Slustky matrix of compensated changes is non-negative, see Mas-Colell (1995, 937):

$$|\mathcal{S}(\mathbf{P}_x, \mathbf{P}_y, \mathbf{I})| = \left.\frac{\partial \bar{x}(\mathbf{P}_x, \mathbf{P}_y, \mathbf{I})}{\partial \mathbf{P}_x}\right|_{\bar{\mathbf{I}}} \left.\frac{\partial \bar{y}(\mathbf{P}_x, \mathbf{P}_y, \mathbf{I})}{\partial \mathbf{P}_y}\right|_{\bar{\mathbf{I}}} - \left(\left.\frac{\partial \bar{y}(\mathbf{P}_x, \mathbf{P}_y, \mathbf{I})}{\partial \mathbf{P}_x}\right|_{\bar{\mathbf{I}}}\right)^2 \geq 0,$$

The determinant of Slustky matrix of substitution effects in this case is given by:

$$|\mathcal{S}(\mathbf{P}_x, \mathbf{P}_y, \mathbf{I})| = \bar{y}(\mathbf{P}_x, \mathbf{P}_y, \mathbf{I}) \left(\frac{\partial \mathbb{E}\left[X \,|\, 0 \leq X \leq \mathbf{Z} \right]}{\partial \mathbf{Z}} \right) \begin{vmatrix} -\mathbf{P}_y/\mathbf{P}_x^2 & 1/\mathbf{P}_x \\ 1/\mathbf{P}_x & -1/\mathbf{P}_y \end{vmatrix} .$$

Substitution of the previous expressions shows that the product above equals zero so the substitution effects are not only symmetric but also satisfy the standard conditions of rational demand analysis. (It is interesting to notice that the vanishing of $|\mathcal{S}(\mathbf{P}_x, \mathbf{P}_y, \mathbf{I})|$ may also be deduced from the fact that average demands are homogeneous of degree zero.) Integrability follows from the symmetry and negative definiteness of the Slustky matrix, see Mas-Colell et al. (1995, Section 3H). ■

The previous Lemma shows that the average demand functions under irrational behavior are indistinguishable from the demands under rational choice. Not only do they satisfy the 'Law of Demand' but irrational demands are homogeneous, add up to total income, and have symmetric cross-price effects and a negative definite matrix of substitution effects. Symmetry is perhaps the most surprising outcome because it plays such an important role in preference-based models. Since the way individuals randomize over X and Y in this economy is symmetric (in the sense that they randomize using the same mechanism for both goods), an income-compensated change induces a symmetric response in the budget line along the initial bundle.[10] Still, it is perhaps important to stress once again that the previous results hold without having any underlying preference structure

[10] In Hicksian demands, compensations are traced out along the indifference curve and hence they depend on the concavity of the utility function. Symmetry of cross-price changes is related to the possibility of interchanging the order of taking partial derivatives in the expenditure function. Symmetry in this paper is not related to indifference curves but to the method of randomization between choices.

behind individuals' choices.[11]

The previous Lemmas also suggest that the irrational system of demands can be *rationalized*. That is, despite the fact that choices are not based on a utility function, the average demands $\bar{x}(\mathbf{P}_x, \mathbf{P}_y, \mathbf{I})$ and $\bar{y}(\mathbf{P}_x, \mathbf{P}_y, \mathbf{I})$ can be obtained from the maximization of some utility function subject to (1). The utility function that rationalizes these demands is monotone and its indifference curves are strictly convex to the origin. In Becker's (1962) analysis of a uniform distribution, as we noted in equation (2), $U(X, Y) = XY$ rationalizes random choices.

3 Interior allocations

This section considers the case in which decisions for X and Y are taken simultaneously and the budget constraint (1) can hold as an inequality. In this case, individuals would leave the proverbial $100 bill on the sidewalk once their decisions are made and they are in the interior of the budget set. This form of irrationality is only briefly mentioned in Becker (1962). As it turns out, however, allowing for interior choices does not alter in any fundamental way the structure of demands or the main conclusions of this paper.

The cumulative distribution is now denoted by $F(x, y)$ and the joint probability density function of X and Y is $f(x, y) := \partial^2 F(x, y)/\partial x \partial y$. Let $\mathbf{p} := \mathbf{P}_x/\mathbf{P}_y$ represent the relative price of good X (in terms of good Y) and recall that $\mathbf{Z} := \mathbf{I}/\mathbf{P}_x$ is the maximum quantity

[11] The treatment of goods X and Y is symmetric and the randomization over X does not depend directly on $\mathbf{P}_y$. In an alternative randomization, assume consumers select among feasible X 'knowing' that there is an additional good Y. For example let $\bar{x}(\mathbf{P}_x, \mathbf{P}_y, \mathbf{I}) := \mathbb{E}\,[X \,|\, 0 \leq \mathbf{P}_x X + \mathbf{P}_y Y \leq \mathbf{I}]$, and let $\bar{y}(\mathbf{P}_x, \mathbf{P}_y, \mathbf{I})$ be determined as a residual. Income effects for Y in this case need no longer be positive. The 'Law of Demand,' symmetry, and additional properties are difficult to establish in general for this randomization. The case in which both goods use this randomization is developed in Section 3.

of X that can be purchased at a given income $\mathbf{I}$ and price $\mathbf{P}_x$.

The average demand function for X is given by:

$$\bar{x}(\mathbf{P}_x, \mathbf{P}_y, \mathbf{I}) := \mathbb{E}\left[X \middle| 0 \leq \mathbf{P}_x X + \mathbf{P}_y Y \leq \mathbf{I}\right] = \frac{\displaystyle\int_0^{\mathbf{Z}} \left[\int_0^{\mathbf{p}(\mathbf{Z}-x)} x f(x,y) dy\right] dx}{\Pr\left[0 \leq \mathbf{P}_x X + \mathbf{P}_y Y \leq \mathbf{I}\right]}, \tag{10}$$

with

$$\Pr\left[0 \leq \mathbf{P}_x X + \mathbf{P}_y Y \leq \mathbf{I}\right] = \int_0^{\mathbf{Z}} \left[\int_0^{\mathbf{p}(\mathbf{Z}-x)} f(x,y) dy\right] dx,$$

as the probability that (1) holds. The average demand for Y, $\bar{y}(\mathbf{P}_x, \mathbf{P}_y, \mathbf{I})$, is defined in similar terms.

One way to interpret the double integral in equation (10) is the following. Integration over x is given by the exterior integral over the feasible values of X; that is, over $[0, \mathbf{Z}]$. Integration over y is given by the interior integral and it uses as a limit of integration the feasible values of Y defined once feasibility over X is determined. As a consequence of Fubini's Theorem one can consider the decisions as taking place in any given 'order.' In other words, one can assume that X is randomly selected first and then Y is selected (subject to an updated feasibility condition) or that both decisions are simultaneous.

The main departure from the previous section is that the random choices of X and Y can now lie inside the budget set. Since interior solutions are allowed, average demands are no longer on the the budget line and hence adding up is not applicable. In other words, the total value of both demands will (almost) always be less than total income:

$$\bar{x}(\mathbf{P}_x, \mathbf{P}_y, \mathbf{I})\mathbf{P}_x + \bar{y}(\mathbf{P}_x, \mathbf{P}_y, \mathbf{I})\mathbf{P}_y \leq \mathbf{I}.$$

Despite this departure, the 'Law of Demand' still holds. Additional properties such as the symmetry of cross-price effects, homogeneity, and negative semi-definiteness are also valid under interior allocations:

Lemma 4 (Law of Demand II) *Assume the random choices of irrational individuals can lie inside the budget set (1). Then, for every* $(\mathbf{P}_x, \mathbf{P}_y, \mathbf{I})$, $\bar{x}(\mathbf{P}_x, \mathbf{P}_y, \mathbf{I})$ *and* $\bar{y}(\mathbf{P}_x, \mathbf{P}_y, \mathbf{I})$ *satisfy the 'Law of Demand,' or equation (6). Moreover, price effects are associated with the Slustky equations (7) and (8).*

Lemma 5 (Homogeneity and the Slustky matrix) *For every* $(\mathbf{P}_x, \mathbf{P}_y, \mathbf{I})$, *under the conditions of Lemma 4, average demands* $\bar{x}(\mathbf{P}_x, \mathbf{P}_y, \mathbf{I})$ *and* $\bar{y}(\mathbf{P}_x, \mathbf{P}_y, \mathbf{I})$ *are homogeneous of degree zero in prices and income and the cross-price derivatives are symmetric. Moreover, the Slustky matrix of substitution effects is negative semi-definite.*

The proofs of the previous Lemmas are in the Appendix because they rely on technical aspects that provide no additional insights. The structure of the proofs is similar to the analysis of non-interior choices but the steps involved are more elaborate since the differentiation is now under two integral signs. Having said that, the intuition derived for the analysis of non-interior allocations applies here as well.

4 Some Generalizations

The purpose of this section is to generalize the analysis in several directions. First, we consider distribution functions whose densities are log-concave. Second, we simulate random behavior for a finite number of individuals and study the empirical properties of the derived demand curves. Additional generalizations include an economy with many goods, an economy with many individual-types, and an exchange economy in which income is price-dependent. These generalizations broaden the scope of the analysis just presented.

4.1 Log-concavity and additional models of random choice

For further characterizations, describe equation (7) in terms of elasticities. The Slustky equation can be written as $\boldsymbol{\varepsilon}_{xx} = \boldsymbol{\varepsilon}^*_{xx} - \mathbf{s}_x \boldsymbol{\eta}_x$, with the typical notion of the elasticities: $\boldsymbol{\varepsilon}^*_{xx}$ as the compensated price elasticity of demand, $\boldsymbol{\eta}_x$ as the income elasticity, and with $\mathbf{s}_x$ as the budget share of income spent in good X.[12] Under additional conditions in $F(x)$ one can sharpen the characterization of the average demand for good X:

Proposition 1 (Log-concavity) *Assume choices are non-interior. Assume also that $F(x)$ is a log-concave distribution function (i.e., $\ln F(x)$ is a concave function). Then, $\mathbf{s}_x \boldsymbol{\varepsilon}^*_{xx} + (1 - \mathbf{s}_x) \leq 0$ and $0 \leq \mathbf{s}_x \boldsymbol{\eta}_x \leq 1$.*

Proof. The proof relies on the following fact of log-concave distributions: $0 \leq \partial \mathbb{E}[X|\, 0 \leq X \leq \mathbf{Z}] / \partial \mathbf{Z} \leq 1$, see Goldberger (1983, Appendix A). ■

Proposition 1 applies to log-concave distributions which is a broad category of distribution functions, see Bergstrom and Bagnoli (2005). (For example, the uniform distribution is log-concave as well as the normal distribution and a relatively large list of well-known probability distributions.) Overall, Proposition 1 shows that if the budget share $\mathbf{s}_x$ is

[12] As a consequence of homogeneity of demands and adding up, the following Engel and Cournot aggregation conditions will (almost) always hold even for irrational consumers: $\mathbf{s}_x \boldsymbol{\eta}_x + (1 - \mathbf{s}_x) \boldsymbol{\eta}_y = 1$, $(1 + \mathbf{s}_x) \boldsymbol{\varepsilon}_{xx} + (1 - \mathbf{s}_x) \boldsymbol{\varepsilon}_{yx} = 0$, and $\boldsymbol{\varepsilon}_{xx} + \boldsymbol{\varepsilon}_{yx} + \boldsymbol{\eta}_x = 0$. See Deaton and Muellbauer (1980, 16) and Mas-Colell et al. (1995, Chapter 2) for a derivation of the previous conditions. The properties stated in Proposition 1 are additions to the previous conditions.

small, the average demand will be highly responsive to changes in $\mathbf{P}_x$ (or relatively price-elastic). The increase in total average spending on X is also less than proportional than the increase in income.

Alternative models of random behavior proposed in the literature complement the case considered by Becker (1962). Chant (1963) and Battalio et al. (1987) considered decisions of "impulsive money deciders." These random decisions are given by the amount of (or the fraction of) money spent in each good. The average demand for good X would then be $\mathbb{E}\left[X \mid 0 \leq \mathbf{P}_x X \leq \mathbf{I}\right]$ if total spending is randomly selected. If the fraction of money spent in good X is randomly selected, the average demand would be $\mathbb{E}\left[S_x \mid 0 \leq S_x \leq 1\right] \times (\mathbf{I}/\mathbf{P}_x)$. The first case is equivalent to the model considered so far while the second case gives demands that are equivalent to those derived from a Cobb-Douglas function (or a Stone-Geary linear expenditure demand system in more general situations, see Battalio et al. (1987)). In the second case, as well as in Becker (1962), average demands have functional forms that match those of well-known utility functions.

4.2 A simulation exercise

A key feature in all previous derivations is that demand curves are defined from the aggregation of individual choices. This sub-section explores a simple simulation exercise whose purpose is to determine how 'large' the economy needs to be in order to observe consistent results due to aggregation. Assume non-interior choices. Assume also that $\mathbf{P}_y$ and $\mathbf{I}$ are constant throughout. At a fixed price level $\mathbf{P}_x$ let X^j, $j = 1, ..., n$ be an i.i.d. sequence of uniform random variables on $[0, \mathbf{I}/\mathbf{P}_x]$. Each j represents an individual realization of demand for good X, i.e., $X^j(\mathbf{P}_x, \mathbf{P}_y, \mathbf{I})$. The average demand is:

$$\bar{x}^n(\mathbf{P}_x, \mathbf{P}_y, \mathbf{I}) = \frac{1}{\#n} \sum_{j=1}^{n} X^j(\mathbf{P}_x, \mathbf{P}_y, \mathbf{I}),$$

which, by the *strong law of large numbers*, satisfies $\bar{x}^n(\mathbf{P}_x, \mathbf{P}_y, \mathbf{I}) \rightarrow \bar{x}(\mathbf{P}_x, \mathbf{P}_y, \mathbf{I})$ with $\bar{x}(\mathbf{P}_x, \mathbf{P}_y, \mathbf{I})$ as considered above. Moreover, when prices $\mathbf{P}_x$ change, one can trace an uncompensated demand curve whose constant elasticity, according to equation (2), should be $|\boldsymbol{\varepsilon}_{xx}| = 1$.

In the following simulations $\mathbf{I} = 1$ and $\mathbf{P}_x$ varies from 1 to 2. The results consider two incremental steps. The first is 0.005 and the second 0.05. This means that each individual j has 200 realizations of demand in the first case and 20 realizations in the second case. The average demand is computed for different values of n that range from $n = 1$ to $n = 1000$. In each sample, and for each value of n, a log-log linear regression estimates the elasticity of the demand curve. The number of cross-samples is 500. One sample realization and the corresponding average demands for $n = 1$, $n = 10$, $n = 100$, and $n = 1000$ is depicted in Figure 2. The general results are presented in the Table 1.

Table 1 shows that 'individual' demands are on average negatively sloped with an elasticity consistent with the predicted pattern. The estimates of the elasticity, however, are unreliable when only one individual realization is considered and the sample variation in prices is small. In fact, the elasticity cannot be statistically distinguished from zero in this case. Further, the standard deviation across and between samples is 1.090 and 0.954 respectively. Both estimates suggest that some individuals have a positively-sloped demand curve. Finally, the goodness of fit from the R^2 is, on average, about 5 percent.

Thus, overall, individual demands are not consistent determined.

Table 1. Simulation results.

	Number of 'individuals' aggregated							
	n=1	n=5	n=10	n=25	n=50	n=100	n=500	n=1000
	A. Grid size for $\mathbf{P}_x$ of 200 sample points							
$\|\hat{\varepsilon}_{xx}\|$	1.0086	1.0095	1.0061	1.0005	1.0011	1.0015	1.0007	1.0001
std.err.	(0.349)	(0.100)	(0.067)	(0041)	(0.029)	(0.020)	(0.009)	(0.006)
std.dev.	[0.345]	[0.102]	[0.067]	[0.040]	[0.029]	[0.020]	[0.008]	[0.006]
R^2	0.0396	0.3343	0.5242	0.7423	0.8538	0.9219	0.9834	0.9917
	[0.027]	[0.054]	[0.043]	[0.026]	[0.014]	[0.008]	[0.001]	[0.0008]
	B. Grid size for $\mathbf{P}_x$ of 20 sample points							
$\|\hat{\varepsilon}_{xx}\|$	1.101	0.9901	0.9966	0.9977	0.9992	1.0013	0.9998	1.0002
std.err.	(0.954)	(0.276)	(0.188)	(0.118)	(0.083)	(0.058)	(0.026)	(0.018)
std.dev.	[1.090]	[0.297]	[0.208]	[0.125]	[0.085]	[0.061]	[0.027]	[0.018]
R^2	0.0562	0.3586	0.5497	0.7633	0.8672	0.9316	0.9857	0.9928
	[0.109]	[0.160]	[0.146]	[0.075]	[0.043]	[0.022]	[0.004]	[0.002]

Note: The number of cross-samples is 500. The elasticities are estimated using a linear fit to log-$\bar{x}^n(\mathbf{P}_x, \mathbf{P}_y, \mathbf{I})$ and log-$\mathbf{P}_x$. The average value of the standard errors across samples is in parentheses. The cross-sample standard deviation for the estimate of the elasticity of demand and the R^2 is in brackets.

Consider the two cases in which $n = 5$. In these cases, the goodness of fit increase to over 30 percent and the estimates of the elasticity of demand become (statistically) close to $|\hat{\varepsilon}_{xx}| = 1$. When the sample variation in prices is 200, and $n = 10$, the across sample standard deviation for the elasticity of demand is about 0.06. The goodness of fit and the statistical significance of the estimates also suggest that average demands are precisely estimated. When only 20 sample points for $\mathbf{P}_x$ are considered, a similar conclusion follows if the aggregation takes place for 50 individuals. As expected, with $n = 500$ or $n = 1000$,

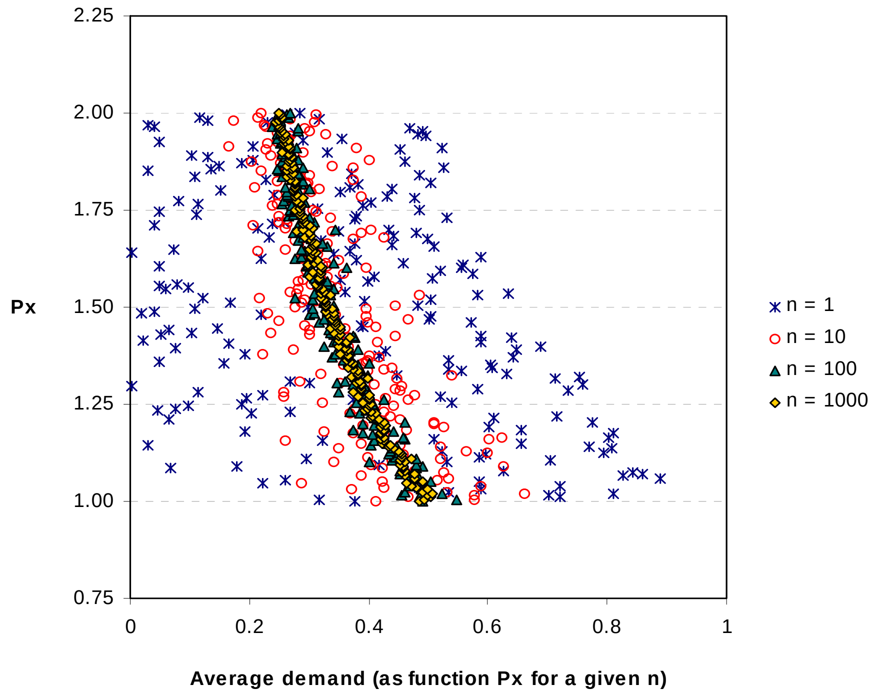

Figure 2: Average uncompensated demand curves aggregated over different number of 'individual' demands. There are 200 demand realizations for each individual. The realization depicted corresponds to one sample from those in Table 1(A).

the goodness of fit is well over 98 percent and the elasticity is precisely estimated up to three digits.

In conclusion, Table 1 shows that with enough price variation, the number of individuals needed to observe a consistent demand curve is small, $n = 10$. With fewer price variations, this number is larger. A market size of $n = 50$ individuals, however, do not seem excessive. That is, while the previous analyses rely on a large number of individuals, numerical approximations with fewer individuals still give support to the main theoretical conclusions of the analysis.

4.3 Many goods

This sub-section of the paper considers a generalization with M goods and non-interior demands. As noted above, we can relax the assumption of non-interior choices without major changes to the basic structure of the model. The budget constraint is now given by $\sum_{i=1}^{M}\mathbf{P}_iX_i =: \mathbf{P}'X = \mathbf{I}$, and so choices $X := (X_1, X_2, ..., X_M)$ depend on a vector $\mathbf{P} := (\mathbf{P}_1, \mathbf{P}_2, ..., \mathbf{P}_M)$ and $\mathbf{I}$. Assume individuals first draw the value for X_1 in a support $[0, \mathbf{I}/\mathbf{P}_1]$. Suppose $\tilde{X}_1$ is realized. Then, individuals draw X_2 in a support $[0, (\mathbf{I} - \mathbf{P}_1\tilde{X}_1)/\mathbf{P}_2]$. Let $\tilde{X}_2$ be the realization for X_2. Choices for X_3 lie in $[0, (\mathbf{I} - \mathbf{P}_1\tilde{X}_1 - \mathbf{P}_2\tilde{X}_2)/\mathbf{P}_3]$, and so on and so forth. The probability a given set of choices is feasible can be written as:

$$\Pr\left(\mathbf{P}'X = \mathbf{I}\right) = \int_0^{\frac{\mathbf{I}}{\mathbf{P}_1}} ... \int_0^{\frac{\mathbf{I}}{\mathbf{P}_M} - \frac{1}{\mathbf{P}_M}\sum_{i=1}^{M-1}\mathbf{P}_i x_i} f(x_1, ..., x_M)\left(dx_1 \times ... \times dx_M\right),$$

with good M given as a 'residual' once spending on all other goods is determined. Demands can be written as:

$$\bar{x}_i(\mathbf{P}, \mathbf{I}) = \frac{\int_0^{\frac{\mathbf{I}}{\mathbf{P}_1}} ... \int_0^{\frac{\mathbf{I}}{\mathbf{P}_M} - \frac{1}{\mathbf{P}_M}\sum_{i=1}^{M-1}\mathbf{P}_i x_i} x_i f(x_1, ..., x_M)\left(dx_1 \times ... \times dx_M\right)}{\Pr\left(\mathbf{P}'X = \mathbf{I}\right)},$$

for $i = 1, ..., M$. As evident from the previous expression, demands are homogeneous and add up to income. Also, notice that all goods are normal on average. Similarly, the total own price change is negative as it follows from the inspection of the equation above. Either $\mathbf{P}_i$ is in the denominator or if it is in the numerator it is preceded by a negative sign. In either case, an increase in $\mathbf{P}_i$ reduces the set of feasible allocations.

4.4 Many individual-types

So far, individuals randomize using a single distribution function $F(x)$. One can generate a class of probability distributions for many types of irrational individuals in the following way. Assume the distribution function $F(x)$ for non-interior solutions considered above depends on a parameter θ by $F(x;\theta)$.[13] Let the support of θ be $\boldsymbol{\Theta}$ and let $G(\theta)$ be a cumulative probability function. Then,

$$F'(x) = \int_{\boldsymbol{\Theta}} F(x;\theta) G(d\theta),$$

is also a distribution function. The mean right-truncated formula is then:

$$\bar{x}(\mathbf{P}_x, \mathbf{P}_y, \mathbf{I}) = \mathbb{E}_{\Theta}\left[\mathbb{E}\left[X | \, 0 \leq X \leq \mathbf{I}/\mathbf{P}_x, \Theta = \theta\right]\right] = \frac{\displaystyle\int_{\boldsymbol{\Theta}} \int_0^{\mathbf{I}/\mathbf{P}_x} x f(x;\theta) G(d\theta) dx}{\displaystyle\int_{\boldsymbol{\Theta}} F(\mathbf{I}/\mathbf{P}_x;\theta) G(d\theta)},$$

which will preserve the properties considered thus far.

Some of the many 'types' we can consider include rational and irrational individuals. Demands for rational individuals will in general depend on prices and income. As long as demands are downward sloping, adding rational behavior will not change the qualitative properties of aggregate demand. One can also allow imitators and selection among groups as considered, for example, by Conlisk (1980). If the demand of imitators tracks pre-determined conventions, and these conventions satisfy some weak form of rational

[13] One simple interpretation for $F(x;\theta)$ is that the random choices are not 'stable.' Individuals may sometimes randomize using $F(x;\theta)$ and some other times by using $F(x;\theta')$ with $\theta \neq \theta'$. Both cumulative distribution functions can be very distinct. One can also consider θ as an index of first order stochastic dominance and impose that $F_{\theta}(x;\theta) \leq 0$ for all x (with strict inequality somewhere). This implies that the average demand is increasing in θ.

behavior, one may observe rational and many irrational types coexisting in any given market.[14]

4.5 An exchange economy

This last sub-section studies an exchange economy and a competitive equilibrium. In contrast to the previous sections, income depends on a given endowment and the prices associated with these endowments. The individual endowment of good X is randomly assigned and it is defined as a random variable X_0. The endowment of good Y is Y_0. Endowments are drawn from a distribution function $F_0(x_0, y_0)$ whose density is $f_0(x_0, y_0)$. (Alternative assignments are obviously possible but they will not alter the significance of the results.) There is a given aggregate endowment of good X, $\mathbf{X}$. The aggregate endowment of good Y is in turn $\mathbf{Y}$ but attention is restricted to X by virtue of Walras' Law which holds as long as allocations are non-interior.

The random assignment of the endowment of good X is given by: $\mathbf{X} = \mathbb{E}_0[X_0] := \int_0^\infty \int_0^\infty x_0 f_0(x_0, y_0)\,(dx_0 \times dy_0)$. The individual budget set is $\mathbf{p}X + Y \leq \mathbf{p}X_0 + Y_0$.

As in the case of non-interior solutions, assume that X is randomly determined subject to $\mathbf{p}X \leq \mathbf{p}X_0 + Y_0$. Also, as before, once X is determined, the demand for Y is given as a residual, $Y = \mathbf{p}(X_0 - X) + Y_0$. A *competitive market equilibrium* in this case is a relative

[14] There are many more possible generalizations that we have omitted to keep the paper within reasonable length. For example, consider an alternative randomization. Before, demand functions are defined only for cases in which X is feasible (i.e., if X does not exceed $\mathbf{I}/\mathbf{P}_x$). Assume there is a positive number $\mathbf{z} \leq \mathbf{I}/\mathbf{P}_x$ such that individuals receive $\mathbf{z}$ if realizations are not feasible. The demand curve in this case is $\bar{\bar{x}}(\mathbf{P}_x, \mathbf{P}_y, \mathbf{I}) = \Pr(0 \leq X \leq \mathbf{I}/\mathbf{P}_x)\bar{x}(\mathbf{P}_x, \mathbf{P}_y, \mathbf{I}) + (1 - \Pr(0 \leq X \leq \mathbf{I}/\mathbf{P}_x))\mathbf{z}$, with $\bar{x}(\mathbf{P}_x, \mathbf{P}_y, \mathbf{I})$ as considered before. In this case, we can also derive the 'Law of Demand' because $\Pr(0 \leq X \leq \mathbf{I}/\mathbf{P}_x)$ is decreasing in $\mathbf{P}_x$. Separately, this paper treats X and Y as deterministic outcomes. To consider the demand for a risky project, assume that a demand of X has as a payoff $\tilde{X} = zX$ with z as some standard random variable that represents the return for each dollar of X invested. Assets can then be treated using the framework developed above. For instance, irrational individuals will demand more risk as their income increases as if they had decreasing absolute *risk aversion*.

price $\mathbf{p}$ that ensures that the market for good X clears; i.e., $\mathbf{p}\left[\bar{x}(\mathbf{p})-\mathbf{X}\right]=0$.

Recall that the average aggregate demand for X is $\bar{x}(\mathbf{p})$ with $\bar{x}(\mathbf{p})=\mathbb{E}[X|0\leq X\leq X_0+Y_0/\mathbf{p}]$ or

$$\bar{x}(\mathbf{p})=\int_0^\infty\int_0^\infty\left[\int_0^{x_0+y_0/\mathbf{p}}x\left(\frac{f(x)}{F(x_0+y_0/\mathbf{p})}\right)dx\right]f_0(x_0,y_0)\,(dx_0\times dy_0)\,.$$

This demand function is well-behaved in the sense that $\bar{x}(\mathbf{p})$ is continuous and homogeneous in prices. Also, demands for X and Y satisfy Walras' Law (as solutions are non-interior). More importantly, the aggregate demand $\bar{x}(\mathbf{p})$ is negatively sloped so there is a unique *competitive equilibrium*:

Corollary 1 (Competitive equilibrium) *Let* $[\bar{x}(\mathbf{p})-\mathbf{X}]$ *be the excess demand function for good* X*. Then,* $\partial\left[\bar{x}(\mathbf{p})-\mathbf{X}\right]/\partial\mathbf{p}\leq 0$ *and so there exists a unique competitive equilibrium in an exchange economy with irrational individuals.*

The Corollary is proved in the Appendix. Corollary 1 shows that the aggregate demand for X is negatively sloped. This implies that the set of equilibrium prices in an exchange economy with irrational individuals is not arbitrary. This result contrast with the well-known Sonnenschein-Mantel-Debreu Theorem in preference-based demand theory. In essence, individual properties of demand functions do not necessarily lead to aggregate behavior consistent with the 'Law of Demand.' In preference-based demand theory, continuity, Walras' Law, homogeneity of degree zero, and the boundary behavior when prices are near zero are preserved by aggregation but they are not sufficient to ensure uniqueness and stability of a competitive equilibrium. The central difference with the Sonnenschein-

Mantel-Debreu Theorem is the nature of aggregation. As recognized by the *statistical approach* to aggregation initiated by Hildenbrand (1983) and Grandmont (1992), certain forms of aggregation may restore nice properties in general equilibrium models. The case studied here also relies on statistical aggregation. The conditions associated with Corollary 1 rely on the fact that income changes affect only the upper limit of integration, i.e., 'individual' demands do not depend on income.[15]

Finally, we can consider decisions in the interior of the budget set as before. The only statement that would be lost is Walras' Law because, by definition, Walras' Law results from the aggregation of individual budget constraints. Equilibrium in the market for good X would be given by a price $\mathbf{p}$ which ensures $\mathbb{E}[X|X \leq X_0 + (Y_0 - Y)/\mathbf{p}] = \mathbf{X}$. Equilibrium in this market, however, would fail to ensure that the market for good Y is also in equilibrium.

5 Concluding remarks

The relationship between random choice and economic principles uncovered by Becker (1962) and generalized here has theoretical importance because it shows that demand functions can be derived without the use of familiar assumptions such as a rational preference order or a utility function.[16] But, is (ir)rational behavior *important*?

[15] The Sonnenschein-Mantel-Debreu Theorem depends on differential income effects. Hildenbrand (1983) shows that if the income distribution has a downward sloping density function, the aggregate income effect will be positive and the 'Law of Demand' will be satisfied. An alternative but related mechanism was considered by Grandmont (1992). In Grandmont (1992), aggregate budget shares are insensitive to price changes so market demands behave as if derived from an aggregate Cobb-Douglas utility function. This ensures a positive aggregate income effect. Additional studies of statistical aggregation include Kneip (1999), Hildenbrand and Kneip (2005), Giraud and Quah (2006), and Quah (1997). Maret (2006) presents a recent overview of the statistical approach to aggregation.

[16] Many previous discussions have addressed the role of utility functions in demand theory, see, e.g., Stigler (1950). McKenzie (1957) considered a 'demand theory without a utility index.' However, this

This question has been a subject of repeated inquiry in economics and there is no intention to provide definite answers here. The following remarks employ insights from the model proposed here to shed light on familiar questions in which irrationality is at the core. We present brief remarks in four critical areas: the assumption of rationality, the role of preferences in economic behavior, aggregation and individual irrationality, and irrationality and welfare.

The assumption of rationality: One fundamental issue raised by random choice is the extent to which one should judge behavioral assumptions by their descriptive or predictive capacity (see Friedman (1953) for an extended discussion that favors the assumptions' predictive ability). Because people are not always fully rational or fully irrational, both are extreme descriptions of human behavior. From a predictive point of view, however, the capacity to analyze changes in economic parameters in rational demand theory and in random choice models is almost the same. The main substantial difference is that, under random choice, goods are always normal. The question of whether or not one can find a random choice model that allows for inferior goods and still preserves the other properties of demand functions remains unresolved.

The nature of irrational choices in this paper is special, perhaps more so than any other form of irrational behavior. But the model proposed in this paper shows that the most basic predictions in demand theory need not be tied to maximizing or complex reasoning. This point is important because a view based on unboundedly rational, self-

approach gives an *alternative* way to represent a utility function, see Newman and Read (1958). Some classical economists, including Cournot, studied demand functions directly and there were some proposals thereafter, including Gustav Cassel's, to employ demand functions without a utility substructure, see, e.g., Stigler (1950, 390). Knut Wicksell objected Cassel's claims, but as Stigler (1950, 391) notes, "Wicksell did not meet the substantive claim of Cassel that it was possible to start directly with demand functions and that the utility added no information of the nature of these functions."

maximizing, and efficient decision makers has been perennially criticized because it is at odds with modern psychological theories of human behavior. (See Friedman (1953, section V) for some earlier remarks about these criticisms and see Becker (1962) for additional discussions.)

The role of preferences: There is another closely related aspect that the assumption of random choice highlights: the relative importance of preferences for understanding economic behavior. In standard rational models, decision making is tossed under an all-encompassing notion of preferences which are determined outside of the model. This suggests that understanding how choices are made in economic problems is central for economic theory. The model proposed here suggests otherwise. Theorem 1, uses a primitive basis of choice.

There are two further aspects to consider. First, as first pointed out by Becker (1962), models of random choice suggest that the basic properties of demand functions are primarily consequences of *opportunity sets* and not exclusively an outcome of the preference ordering over consumption goods or the psychological and neurological process of reasoning involved in decision making. In this viewpoint, preferences and rationality are just convenient, though not essential, assumptions that help examine the response to changes in prices and income. In the words of Stigler and Becker (1977, 76), "the economist continues to search for differences in prices or income to explain any differences or changes in behavior."

The second aspect to consider is that an analysis based on prices and income alone may be too narrow for some economic problems. In many settings, personal motivations and the process of reasoning involved in decision making are fundamental. The understanding of how complex human decision making is has drastically expanded in recent decades. The interaction between economics and psychology has fruitfully lead to mutual learning and important revisions in economists' views of individual behavior. Behavioral economics has demonstrated important systematic decision making biases and has successfully identified a long list of cognitive and psychological elements that are relevant for many economic decisions.[17] (See Kahneman and Tversky (2000) and Rabin (1998) for important discussions.)

The cognitive limits on individual rationality are clearly important. The methods used by behavioral economics are also revealing.[18] For obvious reasons, however, understanding individual behavior is not informative or useful in the model discussed here. For example, the fact that demand curves are downward sloping *on average* does not imply that 'individual' demand curves will behave in the same way. With positive probability, a set of individuals will violate the 'Law of Demand,' (see, e.g., Table 1). After all, individuals do not act rationally and there is no presumption that all individuals will behave in the same way. Any inference based on departures from the 'Law of Demand' from a subset of individuals would not be erroneous, but it would be based on an incomplete representation of all the relevant decisions in the economy.

[17] Behavioral economics has identified a series of psychological, cultural, and emotional motives such as fairness, ambiguity, guilt, care, shame, envy, reciprocity, group identity, and so on as aspects that are important in the notion of personal satisfaction in economics. This paper has not taken any position with respect to the meaning of the goods studied. Therefore, we can define goods X and Y as individual or social emotions and still derive implications on the demand for emotions.

[18] The source of decision making biases has been studied in controlled experiments and even in more interesting settings such as neuroscience laboratories. Recently, 'neuroeconomics' has made extensive use of magnetic resonance imaging (fMRI) to monitor the brain's activity during decision making. ('Physionomics' is now also relying on skin conductance measures, heart rate, measures of hormones, study of pupil dilation and so on.) Camerer et al. (2005), Glimcher (2003) and Gul and Pesendorfer (2008) provide additional remarks and considerations on neuroeconomics.

Economists rely on individual behavior in part because *subjective* judgments determine how goods are valued in the neoclassical theory of value, i.e., goods are valued if they are perceived to be scarce. Individual perceptions are central for economic valuations but some controversy exists about just how far one needs to go to break down individual behavior to its fundamental base. According to Alfred Marshall (1890): "economists study the actions of individuals, but study them in relation to social rather than individual life; and therefore concern themselves but little with personal peculiarities of temper and character." In this regard, our analyses of random choice are consistent with demand theory but the predictions are unrelated to subjective desires. This paper derives an implicit notion of scarcity using the statistical idea of *truncation*. A truncation in the probability distribution of random choices makes certain choices not affordable; it makes goods scarce.

Aggregation and individual irrationality: Aggregation plays a crucial role in the models of random choice we discussed. An aggregative view has been long advocated. Marshall (1890) argued that "economists, like all other students of social science, are concerned with individuals chiefly as members of the social organism. As a cathedral is something more than the stones of which it is made, as a person is something more than a series of thoughts and feelings, so the life of society is something more than the sum of the lives of its individual members."

Aggregation regularizes individual behavior and even provides conditions for the existence of a competitive equilibrium when income is price-dependent. The conditions under which aggregation works need to be highlighted because the results are not robust in several dimensions. First, not all measures of central tendency give well-behaved demand curves. Only the mean and the mid-range give aggregates with nice properties. Second, in the model discussed here, each individual is insignificant and interacts with the market and not with other individuals directly. Third, prices are given and are known in all settings, and finally, individual demands do not depend on prices or income and all variation occurs at the extreme (or at the limit of integration). These assumptions are central for Theorem 1 and without them the conclusions drawn here may not necessarily follow. For example, when individuals interact with other individuals, even small departures from individual rationality may generate large aggregate departures from rationality. Irrationality, for instance, may be 'amplified' if individual's misperceptions complement each other, see, e.g., Fehr and Tyran (2004).[19]

Welfare and irrationality: It is a standard practice in economics to derive demand curves from individual optimization and to use these demand functions to infer properties about social or private *optima*. Utility functions and a notion of rational choice are necessary if one wishes to attach welfare significance to individual behavior. Utility functions and optimization, however, suggest that normative and positive views are always intertwined. Choices in this paper are not based on preferences. Therefore, random choice provides a way to discuss positive aspects without having any normative basis.

An example illustrates this separation. Assume individuals make choices inside their

[19] The paper has considered competitive market settings. The point that economic models predict market and not individual behavior has been already made in the literature and verified in some experimental settings, see, e.g., Plott (1986); Plott and Sunder (1988); Gode and Sunder (1993, 1997) –where random choices employ a uniform distribution as in Becker (1962). There is also a series of interactions between economic agents not mediated by markets in which strategic considerations are important. These problems are usually studied by game theory where behavioral aspects have provided important insights, see, e.g., Fehr and Tyran (2004) and Samuelson (2005).

budget set. In a preference-based view, we may rationalize that these individuals have satiated their desires and that these choices are optimal. (Many other rationalizations are possible but equally inappropriate.) Alternatively, instead of rationalizing objective functions directly, we may study choices and what they reveal about preferences. Choices inside the budget set reveal that individuals are irrational (as there is money left on the table). Such irrationality should prompt a welfare-improving response. We can calculate the consumer surplus to show that if the money left on the table is used to purchase additional goods, consumer surplus should increase. This inference is vacuous because a derived measure of consumer surplus has no normative significance under random choice.[20] Individuals in this paper act without pursuing any objective. Thus, we can confidently *predict* how these individuals will behave in response to changes in the economic environment but we cannot say what *ought* to be the behavior of irrational individuals.

[20] See Ariely et al. (2003) for some difficulties associated with the normative significance of market data and see Bernheim and Rangel (2007) for an important discussion on choice-theoretic behavioral welfare economics. Bernheim and Rangel (2007) present a discussion that relates to an analysis of choices derived from Ariely et al. (2003) and to many other decision-based models.

6 Appendix

Proof of Lemma 1. (Second part) Some steps in this proof are analogous to the proof of Lemma 1. Consider the own-price effect in Y using equation (4):

$$\left.\frac{\partial\bar{y}(\mathbf{P}_x,\mathbf{P}_y,\mathbf{I})}{\partial\mathbf{P}_y}\right|_{\bar{\mathbf{I}}} = \left(\left.\frac{\partial(\mathbf{I}/\mathbf{P}_y)}{\partial\mathbf{P}_y}\right|_{\bar{\mathbf{I}}}\right) - \left.\frac{\partial\bar{x}(\mathbf{P}_x,\mathbf{P}_y,\mathbf{I})}{\partial\mathbf{P}_y}\right|_{\bar{\mathbf{I}}}\frac{\mathbf{P}_x}{\mathbf{P}_y} - \frac{\bar{x}(\mathbf{P}_x,\mathbf{P}_y,\mathbf{I})\mathbf{P}_x}{\mathbf{P}_y^2},$$

but since $\left(\left.\frac{\partial(\mathbf{I}/\mathbf{P}_y)}{\partial\mathbf{P}_y}\right|_{\bar{\mathbf{I}}}\right) = \frac{\bar{x}(\mathbf{P}_x,\mathbf{P}_y,\mathbf{I})\mathbf{P}_x}{\mathbf{P}_y^2}$,

$$\left.\frac{\partial\bar{y}(\mathbf{P}_x,\mathbf{P}_y,\mathbf{I})}{\partial\mathbf{P}_y}\right|_{\bar{\mathbf{I}}} = -\frac{\partial\mathbb{E}\left[X|\,0\le X\le\mathbf{Z}\right]}{\partial\mathbf{Z}}\left(\frac{\bar{y}(\mathbf{P}_x,\mathbf{P}_y,\mathbf{I})}{\mathbf{P}_y}\right)\le 0.$$

Notice that $\frac{\partial\bar{y}(\mathbf{P}_x,\mathbf{P}_y,\mathbf{I})}{\partial\mathbf{P}_y} = -\frac{\bar{y}(\mathbf{P}_x,\mathbf{P}_y,\mathbf{I})}{\mathbf{P}_y}$ and $\frac{\partial\bar{y}(\mathbf{P}_x,\mathbf{P}_y,\mathbf{I})}{\partial\mathbf{I}} = \frac{1}{\mathbf{P}_y} - \frac{\partial\bar{x}(\mathbf{P}_x,\mathbf{P}_y,\mathbf{I})}{\partial\mathbf{I}}\frac{\mathbf{P}_x}{\mathbf{P}_y}$ or

$$\frac{\partial\bar{y}(\mathbf{P}_x,\mathbf{P}_y,\mathbf{I})}{\partial\mathbf{I}} = \frac{1}{\mathbf{P}_y} - \frac{\partial\mathbb{E}\left[X|\,0\le X\le\mathbf{Z}\right]}{\partial\mathbf{Z}}\frac{1}{\mathbf{P}_y}.$$

The Slustky equation for Y follows from simple rearrangements. ■

Proof of Lemma 4. This proof relies heavily on the differentiation under the integral sign. To begin the proof, consider the numerator of equation (10). The derivative with respect to $\mathbf{Z}$ is given by a standard application of the Leibnitz rule for differentiation under the integral:

$$\frac{\partial}{\partial\mathbf{Z}}\left(\int_0^{\mathbf{Z}}\left[\int_0^{\mathbf{p}(\mathbf{Z}-x)} xf(x,y)dy\right]dx\right) = \left.\left(\int_0^{\mathbf{p}(\mathbf{Z}-x)} xf(x,y)dy\right)\right|_{x=\mathbf{Z}} + \left(\int_0^{\mathbf{Z}}\frac{\partial}{\partial\mathbf{Z}}\left[\int_0^{\mathbf{p}(\mathbf{Z}-x)} xf(x,y)dy\right]dx\right),$$

where the first term equals zero. This equation implies that the derivative of the numerator in equation (10) with respect to $\mathbf{P}_x$ is given by:

$$\begin{aligned}\left.\frac{\partial}{\partial\mathbf{P}_x}\left(\int_0^{\mathbf{Z}}\left[\int_0^{\mathbf{p}(\mathbf{Z}-x)} xf(x,y)dy\right]dx\right)\right|_{\bar{\mathbf{I}}} &= \left.\left(\int_0^{\mathbf{Z}}\frac{\partial}{\partial\mathbf{P}_x}\left[\int_0^{\mathbf{p}(\mathbf{Z}-x)} xf(x,y)dy\right]dx\right)\right|_{\bar{\mathbf{I}}},\\ &= \left(\int_0^{\mathbf{Z}} xf(x,\mathbf{p}(\mathbf{Z}-x))\left[\left.\frac{\partial\mathbf{p}(\mathbf{Z}-x)}{\partial\mathbf{P}_x}\right|_{\bar{\mathbf{I}}}\right]dx\right),\end{aligned}$$

with $\left[\left.\frac{\partial\mathbf{p}(\mathbf{Z}-x)}{\partial\mathbf{P}_x}\right|_{\bar{\mathbf{I}}}\right] = \frac{1}{\mathbf{P}_y}\left.\frac{\partial\mathbf{I}}{\partial\mathbf{P}_x}\right|_{\bar{\mathbf{I}}} - \frac{x}{\mathbf{P}_y} = \frac{\bar{x}(\mathbf{P}_x,\mathbf{P}_y,\mathbf{I})-x}{\mathbf{P}_y}$. Hence,

$$\left.\frac{\partial}{\partial\mathbf{P}_x}\left(\int_0^{\mathbf{Z}}\left[\int_0^{\mathbf{p}(\mathbf{Z}-x)} xf(x,y)dy\right]dx\right)\right|_{\bar{\mathbf{I}}} = \left(\int_0^{\mathbf{Z}} xf(x,\mathbf{p}(\mathbf{Z}-x))\left[\frac{\bar{x}(\mathbf{P}_x,\mathbf{P}_y,\mathbf{I})-x}{\mathbf{P}_y}\right]dx\right),$$

and the average demand changes according to:

$$\left.\frac{\partial \bar{x}(\mathbf{P}_x, \mathbf{P}_y, \mathbf{I})}{\partial \mathbf{P}_x}\right|_{\bar{\mathbf{I}}} = -\int_0^{\mathbf{Z}} \{x - \bar{x}(\mathbf{P}_x, \mathbf{P}_y, \mathbf{I})\}^2 \left(\frac{f(x, \mathbf{p}(\mathbf{Z}-x))}{\mathbf{P}_y \Pr[0 \le \mathbf{P}_x X + \mathbf{P}_y Y \le \mathbf{I}]}\right) dx \le 0.$$

To derive the total price effect notice that $\dfrac{\partial \mathbf{p}(\mathbf{Z}-x)}{\partial \mathbf{P}_x} = -\dfrac{x}{\mathbf{P}_y}$ and

$$\frac{\partial \bar{x}(\mathbf{P}_x, \mathbf{P}_y, \mathbf{I})}{\partial \mathbf{P}_x} = -\int_0^{\mathbf{Z}} x \{x - \bar{x}(\mathbf{P}_x, \mathbf{P}_y, \mathbf{I})\} \left(\frac{f(x, \mathbf{p}(\mathbf{Z}-x))}{\mathbf{P}_y \Pr[0 \le \mathbf{P}_x X + \mathbf{P}_y Y \le \mathbf{I}]}\right) dx \le 0,$$

while the income effect is:

$$\frac{\partial \bar{x}(\mathbf{P}_x, \mathbf{P}_y, \mathbf{I})}{\partial \mathbf{I}} = \int_0^{\mathbf{Z}} \{x - \bar{x}(\mathbf{P}_x, \mathbf{P}_y, \mathbf{I})\} \left(\frac{f(x, \mathbf{p}(\mathbf{Z}-x))}{\mathbf{P}_y \Pr[0 \le \mathbf{P}_x X + \mathbf{P}_y Y \le \mathbf{I}]}\right) dx \ge 0.$$

Notice that one can write[21]

$$\int_0^{\mathbf{Z}} x f(x, \mathbf{p}(\mathbf{Z}-x)) dx =: \mathbb{E}\left[X \mid 0 \le X \le \mathbf{Z}, Y = \mathbf{p}(\mathbf{Z}-X)\right],$$

which implies $\mathbb{E}\left[X \mid 0 \le X \le \mathbf{Z}, Y = \mathbf{p}(\mathbf{Z}-X)\right] \ge \mathbb{E}\left[X \mid 0 \le \mathbf{P}_x X + \mathbf{P}_y Y \le \mathbf{I}\right]$ in the previous expression so goods are also normal on average. Finally, as before, one can derive a Slustky equation as in equation (7). Since Y is symmetric, there is no need to study this demand independently. ■

Proof of Lemma 5. Homogeneity is satisfied by definition as demands only depend on $\mathbf{p}$ and $\mathbf{Z}$, both are relative quantities. Consider cross-price effects. To save on notation, we write $y(x) := \mathbf{p}(\mathbf{Z}-x)$,

$$\left(\left.\frac{\partial \mathbf{p}(\mathbf{Z}-x)}{\partial \mathbf{P}_y}\right|_{\bar{\mathbf{I}}}\right) = \frac{\bar{y}(\mathbf{P}_x, \mathbf{P}_y, \mathbf{I}) - y(x)}{\mathbf{P}_y}, \text{ and } \left(\left.\frac{\partial \mathbf{p}(\mathbf{Z}-x)}{\partial \mathbf{P}_x}\right|_{\bar{\mathbf{I}}}\right) = \frac{\bar{x}(\mathbf{P}_x, \mathbf{P}_y, \mathbf{I}) - x}{\mathbf{P}_y}.$$

Then,

$$\left.\frac{\partial \bar{x}(\mathbf{P}_x, \mathbf{P}_y, \mathbf{I})}{\partial \mathbf{P}_y}\right|_{\bar{\mathbf{I}}} = \int_0^{\mathbf{Z}} \{x - \bar{x}(\mathbf{P}_x, \mathbf{P}_y, \mathbf{I})\} \left(\frac{f(x, \mathbf{p}(\mathbf{Z}-x))}{\Pr[0 \le \mathbf{P}_x X + \mathbf{P}_y Y \le \mathbf{I}]}\right) \left[\frac{\bar{y}(\mathbf{P}_x, \mathbf{P}_y, \mathbf{I}) - y(x)}{\mathbf{P}_y}\right] dx,$$

[21] An alternative derivation of the marginal distribution in response to a change in $\mathbf{Z}$ is as follows. Let $Z_1 = X$ and $Z_2 = X + Y/\mathbf{p}$. Then, $X = Z_1$ and $Y = \mathbf{p}(Z_2 - X)$. Using the change of variables formula the density function of (Z_1, Z_2) is:

$$f(z_1, \mathbf{p}(z_2 - z_1)) \begin{vmatrix} 1 & 0 \\ -\mathbf{p} & \mathbf{p} \end{vmatrix} = f(z_1, \mathbf{p}(z_2 - z_1))\mathbf{p}.$$

Next integrate over z_2 to obtain the marginal density of $Z_2 = \mathbf{Z}$,

$$\int_0^{\mathbf{Z}} f(z_1, \mathbf{p}(\mathbf{Z} - z_1))\mathbf{p} dz_1, \text{ or } \int_0^{\mathbf{Z}} f(x, \mathbf{p}(\mathbf{Z}-x))\mathbf{p} dx.$$

Additional treatments of the differentiaton under the integral sign considered here are available in Khuri (2002) and Flanders (1973).

in which, once again $y(x)$ has been written as $y(x) = \mathbf{p}(\mathbf{Z} - x)$. Also,

$$\frac{\partial}{\partial \mathbf{P}_x}\left(\int_0^{\mathbf{Z}}\left[\int_0^{\mathbf{p}(\mathbf{Z}-x)} y f(x,y) dy\right] dx\right)\Bigg|_{\bar{\mathbf{I}}} = \int_0^{\mathbf{Z}} \left(\mathbf{p}(\mathbf{Z}-x)\right) f(x, \mathbf{p}(\mathbf{Z}-x)) \left[\frac{\bar{x}(\mathbf{P}_x, \mathbf{P}_y, \mathbf{I}) - x}{\mathbf{P}_y}\right] dx.$$

Once the previous rules are applied more generally:

$$\frac{\partial \bar{y}(\mathbf{P}_x, \mathbf{P}_y, \mathbf{I})}{\partial \mathbf{P}_x}\Bigg|_{\bar{\mathbf{I}}} = \int_0^{\mathbf{Z}} \{y(x) - \bar{y}(\mathbf{P}_x, \mathbf{P}_y, \mathbf{I})\} \left(\frac{f(x, \mathbf{p}(\mathbf{Z}-x))}{\Pr\left[0 \leq \mathbf{P}_x X + \mathbf{P}_y Y \leq \mathbf{I}\right]}\right) \left[\frac{\bar{x}(\mathbf{P}_x, \mathbf{P}_y, \mathbf{I}) - x}{\mathbf{P}_y}\right] dx,$$

with $y(x)$ also given in terms of x. This concludes the first part of the proof as both derivatives are equal.

To study the Slustky matrix, notice that its determinant is given by:

$$|\mathcal{S}(\mathbf{P}_x, \mathbf{P}_y, \mathbf{I})| = \frac{\int_0^{\mathbf{Z}} \{y(x) - \bar{y}\}^2 f dx \int_0^{\mathbf{Z}} \{x - \bar{x}\}^2 f dx - \left(\int_0^{\mathbf{Z}} \{x - \bar{x}\}\{y(x) - \bar{y}\} f dx\right)^2}{\left(\mathbf{P}_y \Pr\left[0 \leq \mathbf{P}_x X + \mathbf{P}_y Y \leq \mathbf{I}\right]\right)^2},$$

which is positive by the Cauchy-Schwarz inequality. ■

Proof of Corollary 1. The proof follows the same differentiation carried above. For instance, it is possible to show that $\partial\left[\bar{x}(\mathbf{p}) - \mathbf{X}\right]/\partial \mathbf{p}$ is equal to:

$$-\int_0^{\infty}\int_0^{\infty}\left[\left(x_0 + \frac{y_0}{\mathbf{p}}\right) f\left(x_0 + y_0/\mathbf{p}\right) - \int_0^{x_0 + y_0/\mathbf{p}} x f(x) dx\right]\left(\frac{y_0}{\mathbf{p}^2}\right)\frac{f_0(x_0, y_0)}{F(x_0 + y_0/\mathbf{p})}\left(dx_0 \times dy_0\right),$$

which is (almost) always negative. ■

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