Preemption Games: Theory and Experiment*

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Abstract

Several investors face an irreversible investment opportunity whose value V is governed by Brownian motion with upward drift and random expiration. The first investor i to seize the opportunity before expiration receives the current V less a privately known cost C_i; the other investors receive nothing. We characterize Bayesian Nash Equilibrium (BNE)for this game, extending previously known results.

We also report a laboratory experiment with 72 subjects randomly matched into 600 triopolies. As predicted in BNE: subjects in triopolies invested at lower values than in monopolies; changes in Brownian parameters significantly altered investment values in monopoly but not in triopoly; and the lowest cost investor in a triopoly usually preempted the others. Contrary to BNE, subjects' markups of target value over cost did not systematically decrease in cost, but did decrease over time, and remained in the neighborhood of a constrained optimum.

Keywords: Real Options, Preemption, Incomplete Information.

JEL codes: G13, D83, C91, C73

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1 Introduction

Early humans were doubtless familiar with first mover advantages: the first group to harvest a grove of ripening figs, or the first to hunt a herd of antelope, ate better than those who moved later. Such advantages persist to the present day, among high tech firms entering a new product niche (for example, internet-savvy cell phones or fuel-cell powered cars) and among academic researchers investigating a new hot topic. The intervening years offer innumerable other examples in which (a) the value of some opportunity fluctuates over time, (b) several different individuals (or teams) choose the time at which to seize it, and (c) the first to do so gains a valuable advantage.

In this paper we study such situations both theoretically and empirically. We formalize them as preemption games, using standard simplifications to put the strategic issues into sharp focus. Section 2 lays out the assumptions, for example that value of the opportunity follows Brownian motion, that it is observed by a known number of rival investors, that each has a privately known avoidable cost of investing, and that the first mover preempts the entire value of the investment. These assumptions reveal the tension between waiting for the opportunity to ripen and moving quickly to be first.

The theoretical results in section 2 build on earlier work on such preemption games, particularly Lambrecht and Perraudin (2003) and Anderson (2003).[1] Players' equilibrium strategies take the form of a threshold value: at any higher value, the opportunity is seized immediately. We are able to characterize symmetric Bayesian Nash equilibria (BNE) for an arbitrary number of rivals, arbitrary cost distributions, and a full range of Brownian parameters.

Using software created expressly for this purpose, we conduct a laboratory experiment in preemption informed by the theory. Section 3 describes the main treatments—Competition (tripololy) vs Monopoly, and High vs Low Brownian parameters—and obtains five testable hypotheses. Section 4 describes other aspects of the laboratory implementation.

Section 5 presents the results. The first three hypotheses fare quite well: the triopoly market structure leads to much lower investment values than the monopoly structure; the Brownian parameters have a major impact in the predicted direction in Monopoly but (as predicted in BNE) not in Competition; and the lowest cost investor indeed is far more likely to preempt than her rivals. The other two hypotheses don't fare as well. We find evidence that subjects use simpler strategies than

[1] Dixit and Pindyck (1994) and other authors remark on the difficulty of competitive theory in this context, and on the difficulty in obtaining evidence in the field. Perhaps expand on this point in the next draft, and also discuss tangentially related theory such as Lones Smith, and John Morgan on timing games.

in BNE—for example, the markup of threshold over cost tends to be about constant rather than decreasing in cost. Within this class of simple strategies, subjects appear to pick strategies close to the (constrained) optimum, and to lose very little payoff relative to the empirical best response.

Following a concluding discussion, we have a series of appendices on mathematical details, econometric details, instructions to subjects, etc.

In the next draft, say more about the companion paper OFA07, and other literature.

2 Theoretical Results

This section analyzes two situations. In the first, called monopoly, a single investor i has sole access to an investment opportunity. In the second, called competition, two or more investors with private information concerning their own costs have access to the same opportunity, and the first to seize it renders it unavailable to the others.

2.1 Monopoly

An investor i with discount rate $\rho > 0$ can launch a project whenever she chooses by sinking a given cost $C_i > 0$. The present value V of the project follows a geometric Brownian motion with drift parameter $\alpha < \rho$ and volatility parameter $\sigma > 0$:

$$dV = \alpha V dt + \sigma V dz, \tag{1}$$

where z is the standard Wiener process. That is, the value follows a continuous time random walk in which the appreciation rate has mean α and standard deviation σ per unit time. At times $t \geq 0$ prior to launching the project, the investor observes $V(t)$ (and previous values $V(s)$ for $0 \leq s \leq t$). If she invests at time t, then she obtains payoff $[V(t) - C_i]\, e^{-\rho t}$. The project is irreversible and generates no other payoffs. Thus, the task is to choose the investment time so as to maximize the expected payoff.

The solution has been widely known since McDonald and Siegel (1986); see Chapter 5 of Dixit and Pindyck (1994, henceforth DP94) and Oprea et al (2007, henceforth OFA07) for more recent expositions. It turns out that the optimal policy takes the form: wait until $V(t)$ hits the threshold

$$V^*_M(C_i) = (1 + w)C_i, \tag{2}$$

then launch immediately. Note that the threshold is proportional to cost, and that the wait option premium $w \geq 0$ is an algebraic function of the volatility, drift and discount parameters σ, α and ρ.

Specifically,

$$w = \frac{1}{\beta - 1}, \;\; \texttt{where} \;\; \beta = \frac{1}{2} - \frac{\alpha}{\sigma^2} + \sqrt{\left[\frac{\alpha}{\sigma^2} - \frac{1}{2}\right]^2 + \frac{2\rho}{\sigma^2}} > 1. \tag{3}$$

Differentiating (3), one can verify that, over the relevant parameter ranges, w is increasing in σ and α and is decreasing in ρ; again see DP94 and OFA07 for details.

2.2 Competition

Now consider the case that each investor has $n \geq 1$ rivals. All investors $i = 1, 2, ..., n+1$ have access to the same investment opportunity, whose value V again evolves according to geometric Brownian motion (1). Each investor i again knows her own cost C_i, but doesn't know the other investors' costs C_j, $j \neq i$. She regards them as drawn independently according to a cumulative distribution function, $H(C)$, with continuously differentiable density function $h(C)$ and support $[C_L, C_U]$. The first investor to launch obtains payoff $[V(t) - C_i]e^{-\rho t}$ and the other investors obtain zero payoff. All this is common knowledge.

As shown in Appendix A.2, there is a unique symmetric Bayesian Nash Equilibrium (BNE) for the game. It is characterized by a monotone increasing function $V^*(C_i)$ that maps the investor's cost into a threshold value, above which she immediately invests. Here we sketch the derivation and offer some intuition.

The objective function takes the form:

$$F(m|C_i, n) = [V^*(m) - C_i] \left[\frac{V}{V^*(m)}\right]^\beta \left[\frac{1 - H(m)}{1 - H(\hat{C})}\right]^n, \tag{4}$$

where m, V and $\hat{C}$ are defined as follows. The natural choice variable is the threshold value, but since $V^*(C_i)$ is increasing, we can more conveniently write the choice variable as $m \in [C_L, C_U]$, interpreted as the cost-type that the investor chooses as her potential "masquerade".

The first factor in the objective function (4) is simply the profit $[V^*(m) - C_i]$ obtained at the time of successful investment. The second factor, $\left[\frac{V}{V^*(m)}\right]^\beta$, accounts for the time cost of delaying investment and the expiration hazard, given that V is the current value of the investment project. Appendix A.1 notes that the monopolist's value function consists of only these first two factors.

The third and final factor, $\left[\frac{1-H(m)}{1-H(\hat{C})}\right]^n$, is the probability that i has the lowest investment cost, conditioned on the fact that none of the n rivals has already invested. That conditioning is reflected in the denominator, where $\hat{C}$ is the cost corresponding to $\hat{V} \geq V$, the "highest peak" so far achieved by the random walk.

The key step in obtaining the BNE comes from the best response property that investor i maximizes (4) at $m = C_i$. It is straightforward to show that the associated first-order condition can be expressed as the following ordinary differential equation (ODE):

$$V^{*\prime}(C_i) = \frac{[V^*(C_i) - C_i]\, V^*(C_i)}{V^*(C_i) - \beta\,[V^*(C_i) - C_i]} \times \frac{nh(C_i)}{[1 - H(C_i)]} \;\; \texttt{subject to} \;\; V^*(C_U) = C_U. \tag{5}$$

Solution of this first-order condition characterizes the symmetric BNE investment timing rule for our preemption game. Like most non-linear ODE's, Equation (5) apparently does not have a solution expressible in terms of standard functions.[2] However, given a specific cost distribution H and specific values of n and β, Equation (5) can be integrated numerically using standard procedures.

Theorem 1 *Let H be a continuously differentiable cumulative distribution function $H(C)$ on $[C_L, C_U]$. Let it be common knowledge among all investors $i = 1, ..., n+1$ that i's cost C_i is drawn independently from H and that the realization is observed only by investor i. Then there is a unique, symmetric Bayesian-Nash equilibrium in which each investor's optimal investment threshold value is the solution to the ordinary differential equation (5).*

A proof is provided in the appendix.

Lambrecht and Perraudin (2003) use somewhat different methods to derive an ODE that is consistent with Equation (5) for the special case of only two investors (duopoly) and H representing a Pareto distribution. Their methods rule out asymmetric BNE. Our method, adapted from Anderson (2003), has several compensating advantages. It covers $n+1 > 2$ investors, and allows us to drop the parametric restriction on H (and the restriction on the modified hazard rate $\frac{C_i[h(C_i)]}{[1-H(C_i)]}$.) It also gives an explicit (not implicitly defined) ODE, and allows a more streamlined and unified analysis of special cases of interest.

2.3 Special Cases and Bounds

Our experiment employs the uniform distribution H on $[C_L, C_U]$. In this case, (5) reduces to

$$V^{*\prime}(C_i) = \frac{[V^*(C_i) - C_i]\, V^*(C_i)}{V^*(C_i) - \beta\,[V^*(C_i) - C_i]} \times \frac{n}{C_U - C_i} \;\; \texttt{subject to} \;\; V^*(C_U) = C_U. \tag{6}$$

[2] The authors would like to thank Robert Jerrard of the Math Department at the University of Toronto for taking a look at this ODE and consulting with us concerning the possibility of an analytical solution.

It is also instructive to consider the special case where the "net rate of discount" $\beta = 0$. (Inspection of Equation (3) reveals that setting investors' discount rate $\rho = 0$ will suffice.) Then the objective function (4) simplifies to

$$F\left(m|C_i, n\right) = \left[V^*(m) - C_i\right]\left[\frac{1 - H(m)}{1 - H\left(\hat{C}\right)}\right]^n. \tag{7}$$

The ODE (5) then collapses to the well-known ODE for bid functions in first price auctions. Thus, with $\beta = 0$, we obtain the isomorphism noted by Milgrom and Weber (1985), among others, between threshold values in preemption games and bids in auctions.

That well-known ODE is linear for the Vickrey (1963) case of a uniform cost distribution. The Appendix verifies for that for uniform H and $\beta = 0$, the ODE has analytic solution

$$V^{**}(C_i) = \frac{nC_i + C_U}{n+1}. \tag{8}$$

The Appendix further shows that solutions to the more general ODE (6) for uniform distributions are concave functions bounded above by the Vickrey solution (8) and and tangent to it at the upper endpoint. Of course, the solutions are bounded below by Marshallian ($NPV = 0$) investment rule $V^0\left(C_i\right) = C_i$, and the upper and lower bounds intersect at the upper endpoint. More precisely,

Theorem 2 *Let H be the uniform distribution on $[C_L, C_U]$, let n be an integer ≥ 1, and let $V^{**}(C_i)$ be given by equation (8). For $\beta > 0$, the Bayesian-Nash equilibrium threshold $V^*\left(C_i\right)$, the solution to (6) on $[C_L, C_U]$, is a concave function bounded above by V^{**}, tangent to V^{**} at C_U, and bounded below by the diagonal function $V^0\left(C_i\right) = C_i$.*

The intuition carries over to the general case of an arbitrary cost distribution. An investor with the highest possible cost faces Bertrand competition: even a single rival will compete away any attempt to increase profit by raising threshold above cost. Therefore such an investor uses the Marshallian investment rule with markup zero. Other investors increase their threshold up to the point that the greater profit when not preempted just balances the greater threat of preemption by other investors or by "Nature." That last threat (i.e., the expiration hazard) disappears when $\beta = 0$, and so the optimal threshold in that case is an upper bound for the more general case.

Of course, the monopolist threshold is another upper bound on the competition threshold. Thus when the cost distribution is uniform, an upper bound (and a good approximation) of the BNE threshold V^* is the function

$$V^U(C_i) = \min\left\{\left[\frac{nC_i + C_U}{(n+1)}\right]; V_M^*\right\}, \tag{9}$$

where the monopolist threshold V_M^* is given by equations (2) and (3). The monopolist threshold binds in (9) at lower cost realizations for high values of β.

2.4 Constant Markup Rules

The BNE strategy is not as complex as some sorts of feasible strategies. For example, it is not contingent on time elapsed, nor on the current value of the Brownian motion (as long as it is below the threshold!), nor on the history observed so far. However, the BNE strategy does depend on a non-linear function of realized cost. It may be too complex for human investors to discover.

Therefore it may be worth analyzing a restriction of the preemption game to simple 1-dimensional strategy spaces. Here each investor chooses a constant additive markup, i.e., a profit aspiriation $k \geq 0$, and sets the threshold $V^R(C_i, k) = C_i + k$.

In this restricted preemption game, the monopolist's value function simplifies to $F_M(V, C_i, k) = (k)\left[\frac{V}{C_i+k}\right]^\beta$, with expected value

$$E_H F_M(V,k) = kV^\beta \int_{C_L}^{C_U} (C+k)^{-\beta} H(C) dC, \tag{10}$$

The first order condition for the optimum is TBA.

With $n \geq 1$ other investors, we seek a symmetric Nash equilibrium markup k^*.[3] Therefore suppose other investors choose markup $k > 0$ and let investor i consider possible deviations $k_i = k + x$ for any real number x. Her objective function is

$$E_H F(x|k,n) = (k+x)\int_{C_L}^{C_U} \left[\frac{C_L}{C+k+x}\right]^\beta [1 - H(C+x)]^n h(C) dC. \tag{11}$$

Imposing the incentive condition that (11) is maximized at $x = 0$ yields a first order condition whose solution characterizes the NE k^*. This will be written out in the next draft, and obtain comparative statics in n, β and H will be derived. Again, we are especially interested in the case of uniform distributions H, and will summarize the results in a theorem.

[3]Since strategies are not contingent on cost type in the restricted strategy space, we can drop the Bayesian label from the equilibrium concept.

Theorem 3 *Let H be a continuously differentiable cumulative distribution function $H(C)$ on $[C_L, C_U]$. Let it be common knowledge among all investors $i = 1, ..., n+1$ that i's cost C_i is drawn independently from H and that the realization is observed only by investor i. In the special case of the strategy space being limited for all investors to only choosing a constant markup level, k, then there is a unique, symmetric Nash equilibrium in which each investor i will set the same constant markup level, k, independent of her actual iid cost draw. In equilibrium, this common value of k will satisfy Equation (??).*

2.5 Numerical Example

We solved equation (6) numerically for triolopoly ($n = 2$) and a uniform cost distribution on $[C_L, C_U] = [50, 80]$ for $\beta = 2.25$ and $\beta = 3.00$, corresponding respectively to markup factors $w = 0.8$ and $w = 0.5$ in equation (3). Figure 1 shows the resulting functions V^*, together with the corresponding monopoly thresholds V_M^*, the upper bounds from (9), an example of a constant markup rule, and the Marshallian threshold.

[Insert Figure 1 about here.] [Policy graphs of numerical solutions to ODE together with a cost line and the Monopoly solution]

Comment on how the figure illustrates Theorem 2, and comment also on the constant markup restriction results.

2.6 Discrete Approximations

Brownian motion is an idealization. As in OFA07, our experiment uses a close binomial approximation of the continuous time process. Specifically, we use a fixed interval $\Delta t = 0.00\bar{3}$ minutes (i.e., 200 milliseconds) for each discrete step of the value path, and three binomial parameters corresponding to those of the Brownian process:

- the step size $h > 0$ of the proportional change in value, i.e., the current value V becomes either $(1 + h)V$ or $(1 - h)V$ at the next step;
- the uptick probability $p \in (0, 1)$, i.e., the probability that the next step is to $(1 + h)V$ rather than to $(1 - h)V$; and
- the expiration probability $q \in (0, 1)$, i.e., the probability that the current step is the last, and the opportunity disappears.

The deviation of the uptick probability p from 0.5, times the distance $2h$ between an uptick and downtick, corresponds to the Brownian drift rate α:

$$\alpha = \lim_{\Delta t \to 0} \frac{(2p-1)\,h}{\Delta t}. \tag{12}$$

The Brownian volatility σ comes mainly from the stepsize h but when p differs appreciably from 0.5 we must also account for binomial variance $p\,(1-p)$. The exact expression is

$$\sigma^2 = \lim_{\Delta t \to 0} \frac{4p\,(1-p)\,h^2}{\Delta t}. \tag{13}$$

OFA07 explains in some detail why the Brownian discount is given by

$$\rho = \frac{-\ln(1-q)}{\Delta t}. \tag{14}$$

3 Treatments and Hypotheses

Two treatment variables allow us to test the major predictions of the model. The first treatment involves the binomial parameters that govern the value process. We fix the time step at $\Delta t = 0.00\bar{3}$ (in minutes) and the step size at $h = 0.03$. The Low parameter vector is $p = 0.524$ and $q = 0.007$, corresponding to option premium $w = 0.5$ and $\beta = 3.0$. The High parameter vector is $p = 0.513$ and $q = 0.003$, corresponding to $w = 0.8$ and $\beta = 2.25$. In OFA07, the same two parameter configurations were labeled Medium A and High respectively. These configurations differ considerably from each other, yet both yield value paths ”in the money” often enough, and jagged enough, to maintain subjects' interest.

The second treatment variable is market structure. In the Monopoly treatment, subjects made investment decisions with no rivals ($n = 0$) and therefore no risk of preemption. In the Competition treatment, subjects competed in triopolies ($n = 2$). Each period the subjects were randomly reassigned to one of three or four separate markets, each with three investors.

Each period each subject's cost was drawn independently from $U[50, 80]$, the uniform distribution with support [50, 80]. Each session began with a 10 period Monopoly block, continued with 25 periods of Competition, and ended with MonopolyII, another 10 period Monopoly block. The data analysis will focus on the Monopoly and Competition blocks; see Appendix A3 for an analysis of MonopolyII data, which generally leads to parallel (but somewhat more diffuse) results.

Figure 1 plots the theoretical benchmarks as markups, i.e., as optimal threshold value less cost. Dotted lines represent Monopoly markup, $V_M^*(C_i) - C_i$, and solid lines are BNE markups in

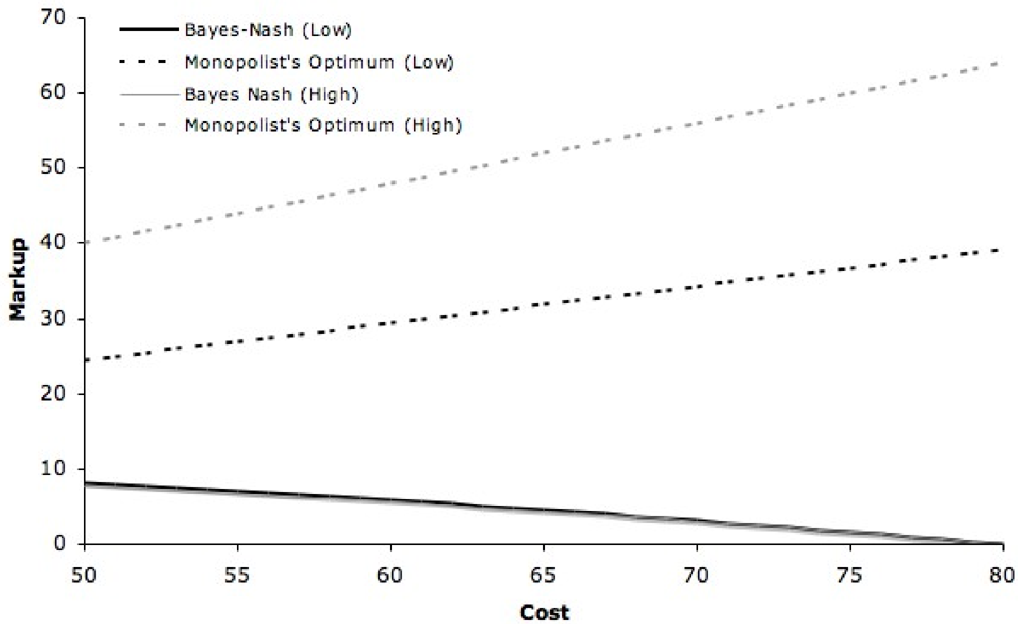

Figure 1: Optimal investment values and numerically estimated Bayesian-Nash strategy functions for each parameter set.

Competition, $V^*(C_i) - C_i$ from the numerical solution to equation (6) with $n = 2$ (triopoly) and the given cost distribution H (uniform on [50, 80]), for $\beta = 3.0$ and 2.25 (Low and High parameters).

Figure 1 shows that, for either parameter vector, the Competitive markups are everywhere much lower than the Monopoly markups. Our first hypothesis is that the markups observed in the experiment will have the same ordering. We express the hypothesis directly in terms of investment value, i.e., the observed value at which an investor chooses to launch a project.

Hypothesis 1 *(Structure) As compared to Monopoly, the Competition treatment significantly reduces investment values.*

Another striking aspect of Figure 1 is that Monopoly line for the High parameter vector is far above the corresponding Low line, while under Competition the two lines are virtually identical. The second hypothesis is that these theoretical orderings will be seen in the experimental data:

Hypothesis 2 *(Parameters) Investment values in the High/Monopoly data significantly exceed those in the Low/Monopoly data, but the data have the same distribution in High/Competition as in Low/Competition.*

Recall that the symmetric BNE strategy $V^*(C_i)$ is increasing in cost C_i. A direct implication is that the investment opportunity is always seized by the lowest cost investor. Allowing for some behavioral noise, we obtain the following cost sorting hypothesis:

Hypothesis 3 *(Efficiency) Under Competition, the most efficient (lowest cost) firm is the one most likely to preempt the others.*

A final observation from Figure 1 is that the markup under Competition is decreasing in costs. The reason is that the slope $V^{*\prime}(C_i) < 1$; indeed, as can be seen from equation (9) and the surrounding discussion, that slope is about 2/3, so the optimal markup slope in Competition is about -1/3.

Hypothesis 4 *(Monotonicity) Under Competition, observed markups are decreasing in cost.*

These hypotheses assume rapid convergence of behavior towards optimum or Bayesian Nash Equilibrium. That assumption implies something that we can easily test: that the observed relationship between cost and investment value is time-invariant.

Hypothesis 5 *(Equilibrium) In each treatment, the observed relationship between cost and investment is stable over time.*

A possible alternative hypothesis is that observed behavior gradually adjusts so that the last few observations are significantly closer to prediction than are earlier observations.

4 Implementation

Experiments were conducted using Investment Timing, the same customized software used in OFA07. Figure 2 shows the user interface. The lighly shaded blue band indicates the cost range, $[50, 80]$, which was held constant throughout the session and announced publicly. The horizontal red line represents the subject's own cost that period; its status as private information was also announced publicly.

The current value of investment, $V(t)$, was represented by a jagged green line that evolved from the right, as on a seismograph. During each period the green value line was initialized at 50 (the lower bound of the cost distribution)[4] and evolved from there according to the binomial parameter

[4]Unlike the experiment reported in OFA07, where the initialization was at realized cost, typically higher than 50.

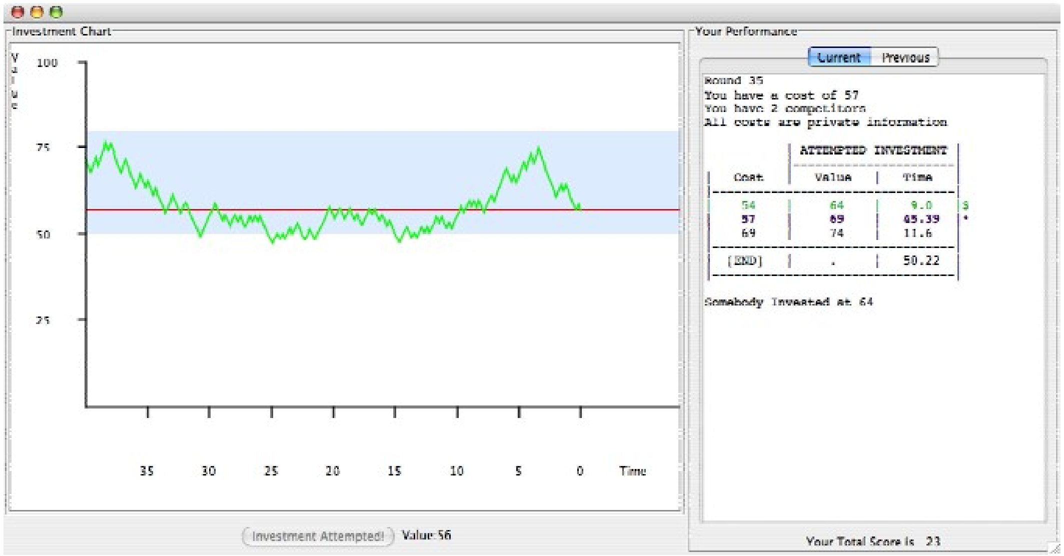

Figure 2: Subject screen under Competition at the end of a round.

vector, High or Low, chosen for that session. The screen rescaled if the value line ever rose out of the given bounds.

Subjects were not allowed to invest when the value line was below their own cost, to prevent negative earnings, nor could they invest after the random ending time. At all other times, subjects could attempt to invest by tapping the space bar at their computer terminal.

In the Monopoly treatment, an investment attempt prior to period end was always successful, immediately netting a subject $V(t) - C_i$ points. Subjects in the Competition treatment were not told whether or when their competitors invested until after the period was over. This semi-strategy method gives us access to more data, while still giving subjects the real-time choice experience that we feel helps them adapt to the stochastic environment.[5]

After the period ended, subjects were told the time each subject in their group attempted investment, the value at which they attempted investment, the costs of each competitor, and the resulting profits: $V(t) - C_j$ to the subject who invested first, and zero to the others. In the Monopoly treatment, of course, there were no other subjects in the group.

All cost draws, value sequences and period endings were made only once for each parameter set, and repeated in all sessions for that treatment. This procedure permits sharper tests of the

[5]Implementing the full strategy method would require us to constrain the strategy space, e.g., to a choice of threshold $V(C)$, excluding a priori non-stationary and other sorts of strategies that subjects might use. Unfortunately our semi-strategy method censors choices in periods that end before a subject attempts to invest.

	Parameters			
Treatment	**h**	**p**	**q**	**Replications**
Low	0.03	0.524	0.007	36 subjects
High	0.03	0.513	0.003	36 subjects
	Blocks/Periods			
Treatment	**1-10**	**11-35**	**36-45**	
Low	Monopoly	Competition	Monopoly	
High	Monopoly	Competition	Monopoly	

Table 1: Experimental Design.

hypotheses. For example, a different sequence of cost draws in Competition than in Monopoly might itself have a significant impact and cloud the inference on the impact of the structure treatments. Moreover, the realized cost distributions are nearly identical across parameter sets. In one session under High parameters, a software malfunction during period 30 (towards the end of the Competition block) lead to 3 missing periods which have been dropped from the dataset.

Experiments were conducted at the University of California, Santa Cruz using inexperienced undergraduate subjects recruited using an online database. Subjects were paid a $5 showup fee. Earnings during the experiment were expressed in "points." Subjects were paid 5 cents for each point earned in monopoly rounds. Subjects were paid 15 cents for each point earned under competition rounds so that the expected per point earnings from an investment decision would be maintained at 5 cents across treatments. Subjects earned an average of $19.56. Table 1 summarizes the design.

5 Results

Data analysis must account for two complications. First, the method of inducing impatience produces random ending times, so observed investment values have random right hand censoring. Therefore any estimate using observed investment values alone would suffer downward bias. Second, because subjects cannot invest at values below their costs, the data are left truncated at cost.

To cope with these complications, we generate product limit (PL) estimates of the empirical distribution function of investment values from each treatment. As explained in the Appendix

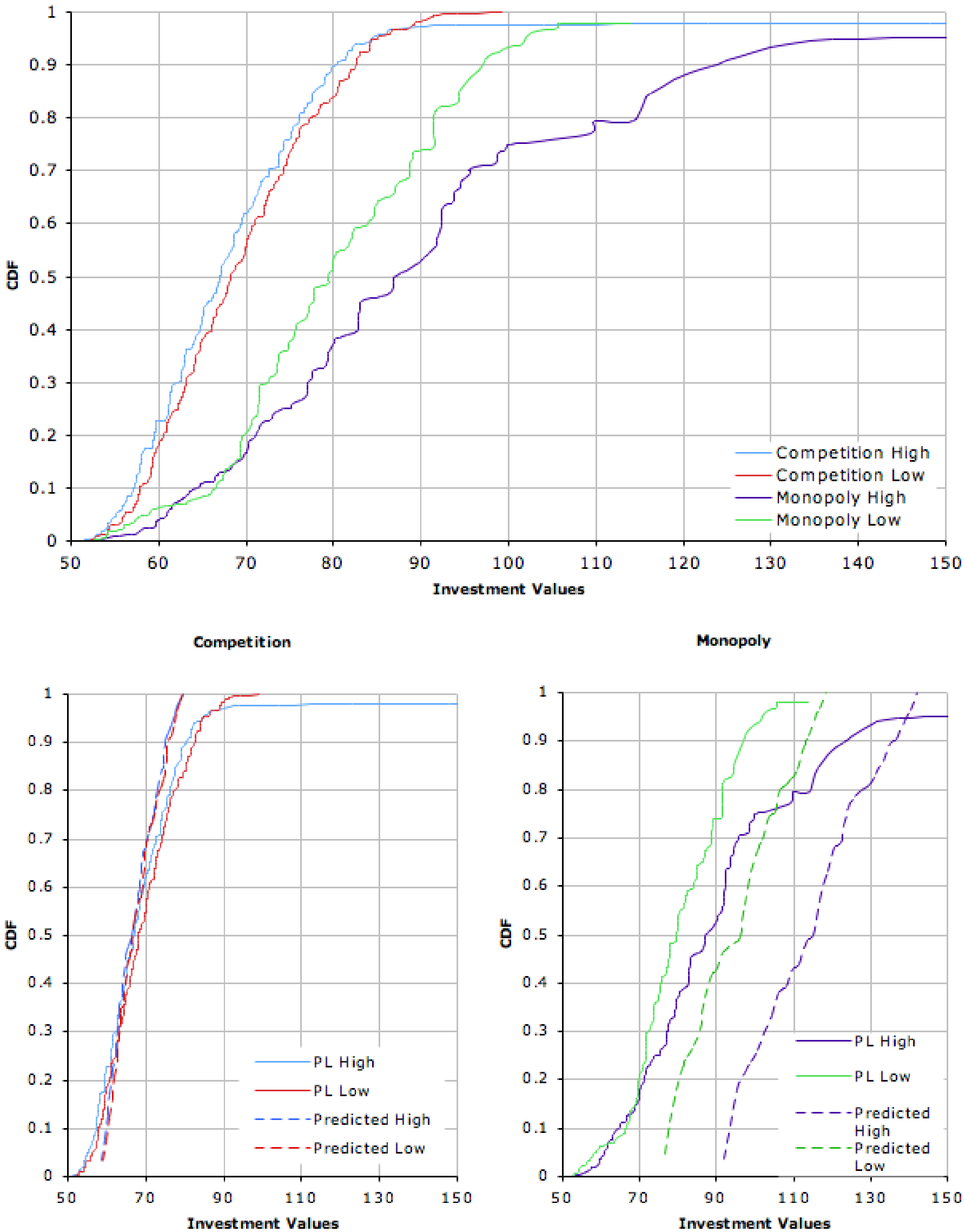

Figure 3: Product limit CDFs.

B, the PL procedure provides maximum likelihood non-parametric estimates of the cumulative distribution function (CDF). The estimates are graphed for each treatment in Figure 3.

The top panel of Figure 3 suggests that investment behavior is considerably different under Competition than under Monopoly. For example, the median investment values (where the graph crosses the horizontal line at 0.5) are about 66 or 67 for both Competition treatments versus about 80 for Low/Monopoly and about 87 for High/Monopoly. The observed ordering seems consistent with the first two hypotheses.

The bottom panels of Figure 3 compare observed distributions (solid lines) with the predicted distributions. The dashed lines indicate the PL estimates for optimal Monopoly behavior according to equations (2, 3) and BNE Competition behavior according to the numerical solutions to equation (6), given the realized cost draws. In the Competition panel, the predictions for High and Low parameters are very close together. The CDFs for observed investment values for High and Low parameters are also close to each other, and only slightly more diffuse than predictions. In the Monopoly panel, the predictions for High parameters are about 15-20 higher (i.e., to the right of) those for Low parameters at each percentile. Above the 20th percentile, the CDFs for observed investment values have the same ordering and about the same spacing above the 80th percentile, but for the most part they fall well below (i.e., to the left of) the theoretical predictions.

5.1 Tests of Treatments

The PL estimates can be compared statistically using a variant of the Mann-Whitney test called the log-rank test; again see Appendix B. That test confirms that the differences between Monopoly and Competition are significant at the one percent level for both High parameters and for Low parameters. As a supplementary test to control for within-subject variance, we also estimate the product-limit mean investment value for each subject under each treatment and market structure. Comparing the populations of estimates using Mann-Whitney tests confirms that investment values are higher under Monopoly than under Monopoly for both Low parameters and High parameters (both with $p = 0.000$). We report these observations as a first finding.

Finding 1 *Consistent with Hypothesis 1, aggregate investment values controlling for cost and for binomial parameters are significantly lower under Competition than under Monopoly. The same is true for individual subjects' investment values.*

Our second hypothesis predicts that investment values will be sensitive to binomial parameters

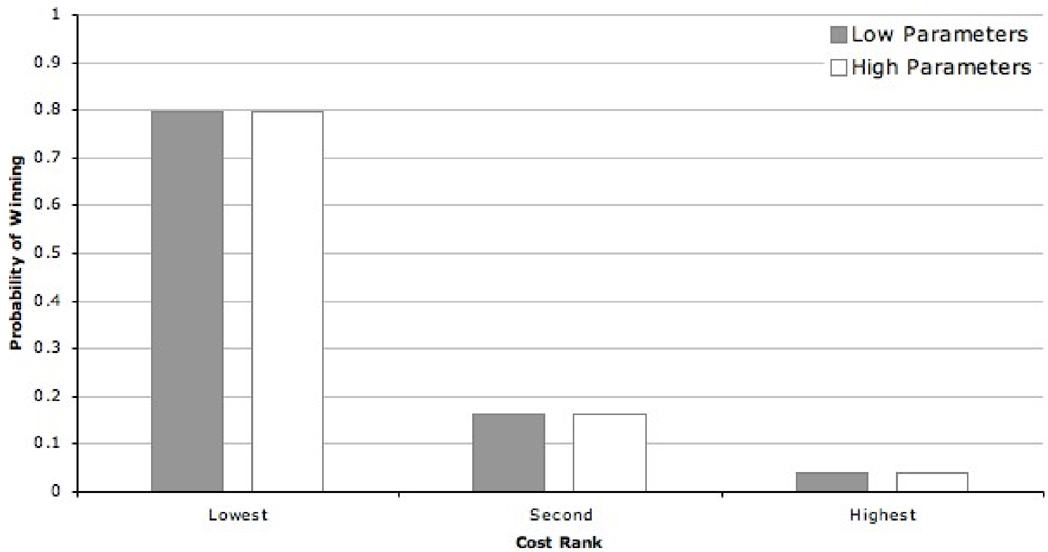

Figure 4: Probability of winning investment as a function of cost rank.

under Monopoly but not under Competition. Comparing CDF estimates across parameter sets seems to support this conjecture. Recall from Figure 3 that investment values are typically larger under High parameters than low parameters in the Monopoly block. This difference is significant at the one percent level according to the relevant log-rank test. Visually one can see that in Competition, the Low investment values exceed High by about 1-3 points at most percentiles (the opposite direction from that one might expect) but the log-rank test indicates that the difference is insignificant ($p = 0.1090$). The same story holds if we compare by-subject PL mean values. Investment values are higher under High parameters under Monopoly (Mann-Whitney $p = 0.0042$) though not under Competition ($p = 0.2460$). Together these test results provide us with a second finding.

Finding 2 *Consistent with Hypothesis 2, investment values in Monopoly periods are significantly larger under High parameters than Low parameters, and there is no significant difference in Competition periods.*

5.2 Cost and Investment in Competition

Our third hypothesis is that the lowest cost investor (the efficient one) will usually preempt her rivals. Figure 4 shows the fraction of times an investment is made by the lowest, middle and highest cost investor for each parameter set. In both treatments the lowest cost investor wins roughly 80

	Low Parameters		High Parameters	
Parameter	(1)	(2)	(1)	(2)
c	0.0053 ± 0.0057	0.0054 ± 0.0057	0.0053 ± 0.0067	-0.0012 ± 0.0057
Elapsed		0.0314*** ± 0.0060		0.0432*** ± 0.0073
θ	0.112*** ± 0.0057	0.1201*** ± 0.04884	0.193*** ± 0.0697	0.2293*** ± 0.0772
Nobs	567	567	472	472

Table 2: Cox regressions on markups

percent of the time while the highest cost investor wins less than 5 percent of the time. Note that these tendencies don't much vary across parameter sets. Equally important, higher cost subjects also tend to preempt lower cost subjects only when costs are very close. On average, the difference between the middle and low cost draws is $(C_U - C_L)/(n+1) = (80-50)/(3+1) = 7.5$ but in periods when a second lowest cost subject preempts the lowest cost subject, the median difference is only 2.5. Likewise, when a highest cost subject preempts the lowest cost subject, the median difference is only 4, as compared to an a priori average of $2 \times 7.5 = 15$. To summarize,

Finding 3 *Consistent with Hypothesis 3, the lowest cost investors usually preempt the other investors and the highest cost investor rarely preempts the other investors.*

Hypothesis 4 predicts that under Competition, the target markup $M = V(C) - C$ is decreasing in the cost of investment C. To test the hypothesis, we use the Cox proportional hazard model, a semi-parametric estimate of marginal effects on the instantaneous probability (hazard rate) of an event. Although proportional hazard models are typically used in time-to-event studies, they are easily adapted to our value-at-event data. The Cox model is especially useful for our data because it makes no assumptions violated by the left hand truncation and right hand censoring. We estimate:

$$h_{ij}(M) = h_0(M)e^{\alpha_i + \beta c_{ij}} \tag{15}$$

where j indexes periods and α_i is allowed to vary with subject i.[6]

Table 2 collects the estimation results (expressed as $exp(\beta)$) are presented in columns (1). The variable, θ, a measure of within subject correlation, is significant in both cases indicating substantial

[6]Because we allow α to vary across subjects, this model can be thought of as a random effects estimator for the Cox model. Models in this family are often called shared frailty models. We use the standard assumption that α_i has the gamma distribution with mean 1.

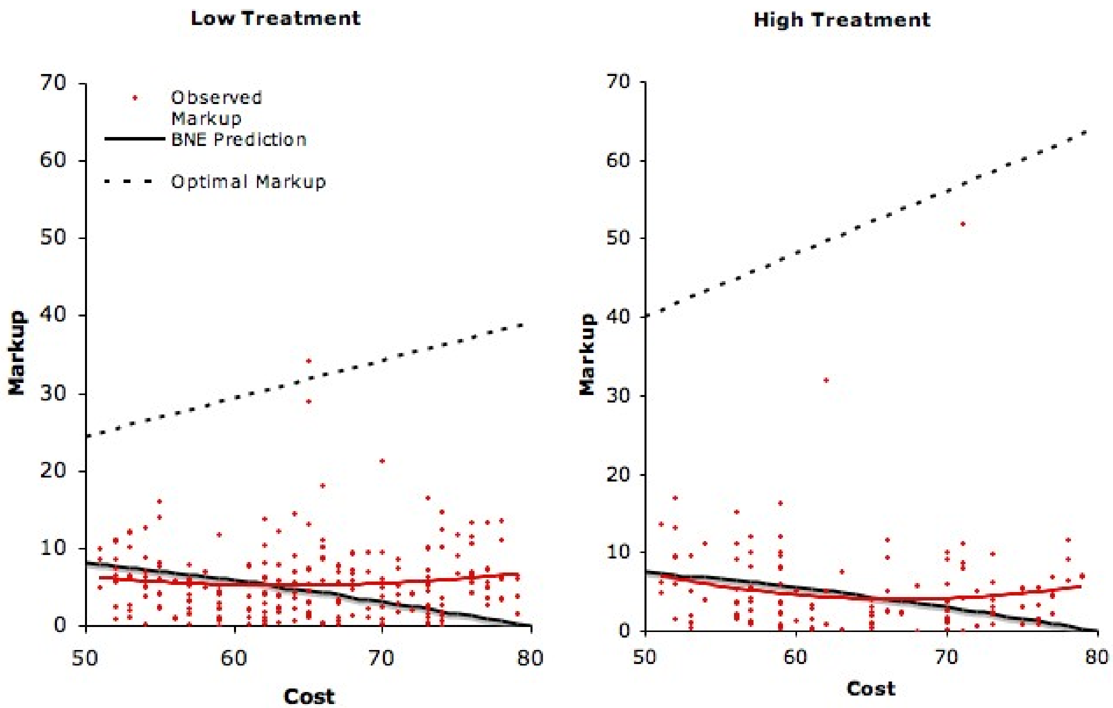

Figure 5: Scatterplots of observed markups and cost by treatment, for the subsample of Competition periods with $\hat{v}$ in the upper quartile. Red lines are quadratic fits to the data. Black lines are BNE predictions. Dotted lines are optimal markups for Monopolists.

between subject heterogeneity. Most importantly, and a bit surprisingly, the coefficent on cost, c, is insignificantly different from zero indicating that markups are, in fact, independent of cost.

To examine this finding more directly, we take a subsample of the data where censoring is not a problem. Note that $\hat{v}$, the maximum value achieved in a given period, is a random variable which is uncorrelated with cost and, because it is unknown to subjects ex ante, is also uncorrelated with the chosen investment threshold. It is however correlated with our ability to observe investment choices: when $\hat{v}$ is particularly high we observe a larger proportion of choices and censoring is rare. Thus, the set of periods with $\hat{v} > 87$, the upper quartile, is an almost unbiased subsample for examining the relationship between markups and costs. This subsample contains 456 observations, only 7 of which are censored.

Figure 5 displays the observed markups in that subsample of Competition. A quadratic fit to the data gives the markup as 7.51 - 1.38*Low - 0.41*Cost + 0.27*Low*Cost + 0.01*Cost*Cost - 0.01*Low*Cost*Cost. The intercept (7.51) is significant at the 1% level, but none of the other

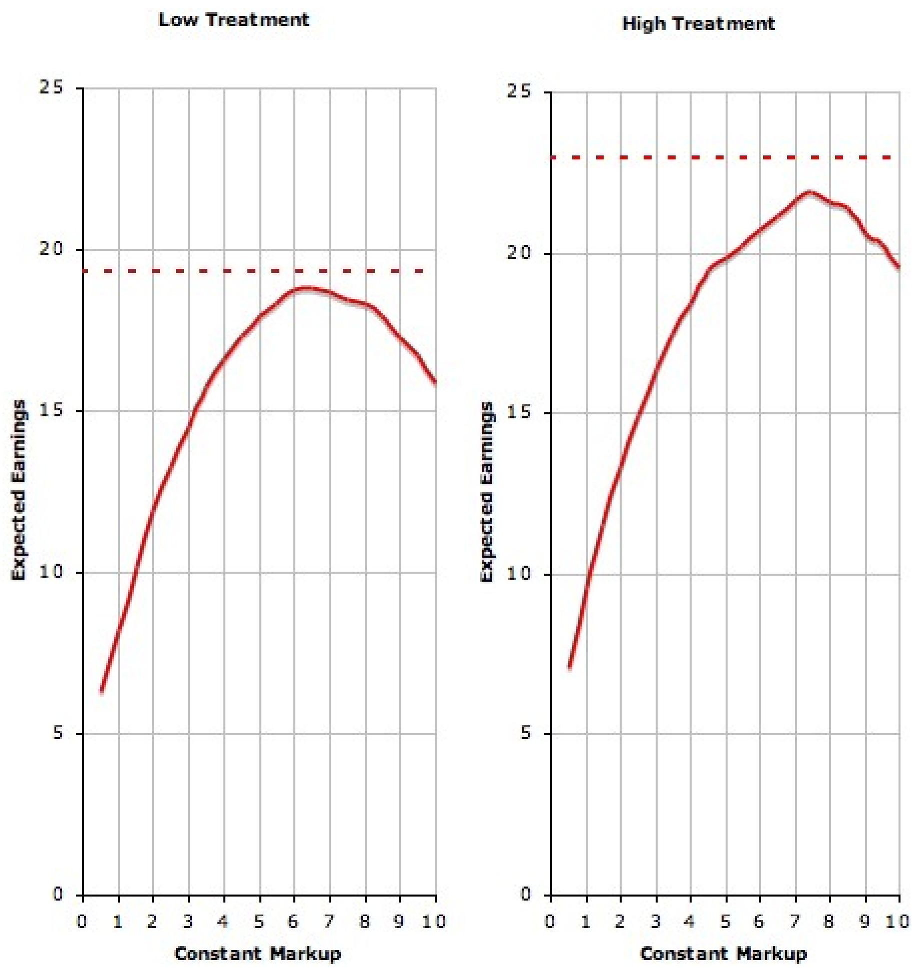

Figure 6: Simulated expected per period earnings for various choices of constant markups played against two rivals using BNE threshold strategies. Earnings are expressed in cents.

coefficients is significant at even the 5% level.[7] There is no graphical evidence of a consistent inverse relationship between cost and markups in either treatment, corroborating the Cox model estimates.

Finding 4 *Contrary to Hypothesis 4, there is no clear evidence that investors' markups decrease in cost in Competition. Rather, markups are approximately constant, independent of cost.*

5.3 Constrained Best Response Markups

How much money do subjects leave on the table when they use a simple constant markup rule in Competition instead of the more complex equilibrium strategy? To investigate, we simulated subjects playing a range of constant markup strategies against two competitors playing BNE strategies over 25 periods. In each of 250 Monte Carlo simulations for each parameter set, we calculated earnings for a subject using 20 different constant markups: 0.5, 1, 1.5, ..., 9.5, 10.0.

The results are collected in in Figure 6. The vertical axis shows average earnings per period. For reference we include a horizontal dashed line showing the average earnings made over these simulations by subjects playing, instead, the approximate Bayes-Nash strategy. Two observations are noteworthy. First, the payoff graphs are unimodal, suggesting unique best responses of about 6.5 for Low parameters and 7.5 for High. Second, the optimal constant markup choice yields payoffs that are rather close to the Bayes-Nash payoffs. The difference in both cases is less than one expected cent per period or $0.25 over the course of the experiment. Thus, although some money is left on the table by choosing a simple constant markup strategy, that amount can be quite small.

The results are similar when the two competitors also use near-optimal constant markups; simulations suggest that with Low (respectively High) parameters, a constant markup of 6.5 (respectively 7.5) is a best response to two rivals using the same strategy. Also, against the empirical distribution of choices, the best constant markup is 7.5 for both High and Low parameters, remarkably close to the intercept of the quadratic fit shown in Figure 5. Against the empirical distribution, the BNE strategy earns very slightly less than the actual average across subjects.

5.4 Dynamics

Two important and related questions remain. First, how do subjects' chosen markups compare to the constrained optimum? Second, are the markups stable over time as in Hypothesis 5, or do they trend towards (or away from) the constrained best response? To investigate both questions at once, we insert into the Cox proportional hazard model the new variable *Elapsed*, representing the number of periods elapsed since the first period of Competition. We estimate

$$h_{ij}(m) = h_0(m)e^{\alpha_i + \beta c_{ij} + \gamma Elapsed_j} \tag{16}$$

[7] Taking the coefficients at face value, markups decrease in cost at about the predicted rate at the low cost end of the distribution, but the relationship reverses sign at higher costs.

Covariate	**Estimates**
Intercept	7.090*** ± 0.625
High	0.744 ± 1.032
Elapsed	-0.111*** ± 0.036
High $\times$ *Elapsed*	-0.102* ± 0.061
σ_u	1.606
σ_ϵ	4.519
ρ	0.112

Table 3: Random Effects regression of markups on elapsed periods.

and report the results in columns (2) of Table 2. We see strong evidence that markups decrease over time, especially with High parameters.

To check robustness and gain further insight, we again look at the subsample of periods whose maximum value ($\hat{v} > 87$) is in the upper quartile. Here we estimate the following random effects regression:

$$M_{ij} = \alpha + \alpha_h \times High + \beta \times Elapsed + \beta_h \times High \times Elapsed + u_i + \epsilon_{ij} \tag{17}$$

where *High* is an indicator variable for the High treatment, u_i is a normally distributed random effect on subject i with mean 0 and variance σ_u and ϵ_{ij} is a disturbance term with mean 0 and variance σ_ϵ. Once again, this model exploits the exogenous and unpredictable nature of $\hat{v}$ to avoid the censoring problem in most of the data. Results are displayed in Table 3. The significant and negative coefficient on *Elapsed* provides further evidence that markups are decreasing in time. The marginally significant negative coefficient on the interaction term *High* $\times$ *Elapsed* provides weak evidence that this dynamic may be stronger under the High treatment than the Low.

Finding 5 *Contrary to Hypothesis 5, subjects in Competition reduce their markups over time. There is weak evidence that this effect is stronger in the High treatment than the Low treatment.*

Table 3 also provides a surprising answer to the question regarding constrained optimality. In the Low treatment subjects begin with a markup of $\alpha = 7.09$ which is insignificantly different from the constrained optimum of 6.5 ($p = 0.345$). In the High treatment subjects begin with a markup of $\alpha + \alpha_h = 7.83$ which is insignificantly different from the constrained optimum of 7.5 ($p = 0.684$).

Finding 6 *Subjects initially use constant markup strategies which represent optimal constrained* *best responses to Bayesian Nash strategies. Moreover subjects lose little in terms of earnings by implementing these constrained strategies.*

What causes lowering of markups over time? To investigate, we estimate a directional learning model in the tradition of Selten and Buchta (1998) and Cason and Friedman (1999). We classify all the different kinds of ex post losses that an investor could suffer, hypothesize that adjustment of markup is proportional to the magnitude of each kind of loss, and estimate the sensitivities to each kind.

One way to lose earnings is to be preempted by a competitor. Denoting the value chosen by the winner as v^w, the loss in this case is $Lose(v^w - c)$ where $Lose$ is a dummy taking a value of 1 when the subject attempts but fails to invest. We hypothesize that this source of loss causes subjects to decrease their markup in future periods.

Another way to lose potential earnings is to successfully invest but to realize that one might have invested successfully at a higher value. Denoting v^s as the value of the second investor to invest, this loss can be measured by $Win(v^w - v^s)$ where Win is a dummy taking a value when the subject successfully invest. We hypothesize that this would cause subjects to increase markups instead.

A third type of loss is to fail to attempt investment at all because the round ends before the subject chooses to attempt. In such case a subject loses $Random(\bar{v} - c)$ where $Random$ is a dummy taking a value of 1 if the subject has the opportunity to invest but all subjects fail to attempt investment.

A final way to lose earnings, though not due to strategic failure, is to miss the opportunity to invest at all. This occurs if the value line never reaches the subject's cost ($\bar{v} < c$). We will represent this with a dummy $NoChance$.

We also include the term $c_t - c_{t-1}$ as a robustness test of our constant markup findings. Under the hypothesis that markups are decreasing in costs, a coefficient on this term should be significantly negative.

Estimating the model is complicated by the fact that we do not observe a subject's investment attempt (or his markup M) in periods where $Random$ or $NoChance$ are equal to one. The Appendix derives a clean way to cope with the problem using weighted least squares.

Estimation results with clustering included at the subject level are presented in Table 4. The coefficient on $Lose \times [v^w - c]$ is significant and negative as expected as is the coefficient on $NoChance$.

Covariate	**Estimates**
$Lose_{t-1}[v^w_{t-1} - c_{t-1}]$	-0.403^{***} ± 0.082
$Win_{t-1}[v^w_{t-1} - v^s_{t-1}]$	0.058 ± 0.041
$Random_{t-1}[\bar{v}_{t-1} - c_{t-1}]$	0.330 ± 0.205
$NoChance_{t-1}$	-0.394^{**} ± 0.157
$c_t - c_{t-1}$	0.019 ± 0.026

Table 4: Estimated adjustment parameters.

All other loss variables are insignificant. Also insignificant is the coefficient on $c_t - c_{t-1}$, giving us further evidence against Hypothesis 4. Thus, the drop in markups over time is driven largely by adjustment in response to preemption. Moreover, subjects adjust their markups downwards after experiencing periods in which the investment opportunity never becomes profitable.

Simulations using the significant parameters predicts substantial decreases in markups over time. Indeed the decrease is somewhat stronger than that observed in our data. Note however that the R^2 on this model is quite low (0.0484) indicating that other factors including subject heterogeneity likely exert a great deal of influence over the pattern adjustment. Moreover, because of our censoring problem our empirical strategy limits us to linear specifications. However our model does indicate that the experience of being preempted and of losing the opportunity to attempt investment significantly influence subjects' markups over time.

Finding 7 *Subjects reduce their target markups in response to being preempted. Moreover subjects target lower markups after periods in which they do not have the opportunity to invest.*

6 Discussion

Our exploration of preemption under uncertainty uncovered several new regularities, empirical as well as theoretical. On the theoretical side, we were able to extend previous work to obtain precise predictions of behavior in competition. In Bayesian Nash equilibrium (BNE), each investor optimally waits until the value of the investment opportunity hits a specific threshold that depends on that investor's private cost. We fully characterized the BNE threshold function for an arbitrary number of competitors and over relevant parameter ranges, and obtained useful approximations. For example, the BNE markup of threshold over cost decreases by about \$1 for each \$3 increase in cost when costs are uniformly distributed.

Our laboratory experiment confirmed several of the theoretical predictions. Observed investment thresholds indeed are much lower in triopoly competition than in monopoly, and changes in the parameters driving the stochastic value process have a strong effect (in the predicted direction) in monopoly but no detectable effect in competition. We also observed the sort of efficiency predicted in BNE: lower cost investors are far more likely to preempt than their higher cost rivals.

Other laboratory findings were not as predicted by the BNE theory. Although there is some weak evidence that triopoly competitors with low cost tend to reduce their markups as cost increases, the overall pattern is more consistent with very simple constant markup strategies.

Equilibrium theory is silent on the dynamics that might lead subjects towards BNE strategies. Our data suggest that competitors reduce their aspirations (i.e., their intended markup) after being preempted by a rival or by the premature expiration of the investment opportunity. However, subjects showed no clear tendency to move closer to the BNE strategies over time, and the data suggest that they would gain little by doing so. Despite using simple (and noisy) strategies, our subjects' earnings were not far from those associated with BNE.

In the next draft, say a little more about behavior in monopoly, noting parallels to OFA07 and explaining any discrepancies. Perhaps also discuss Bertrand competition and common values variants on the preemption game. Be sure to update the bibliography.

References

[1] Berger, P., Ofek, E. and Swary, I., 1996, "Investor Valuation of the Abandonment Option," *Journal of Financial Economics*, 42: 257-287.

[2] Braunstein, Y. and Schotter, A., 1981, "Economic Search: An Experimental Study." *Economic Inquiry*. 19: 1-25.

[3] Cason, T. and Friedman, D., 1999, "Learning in a Laboratory Market with Random Supply and Demand." *Experimental Economics*. 2:277-298.

[4] Carillo, J. and Mariotti, T, 2000, "Strategic Ignorance as a Self-Disciplining Device." *Review of Economic Studies*, 67: 529-544.

[5] Copeland, T. and Antikarov, V., 2003, "Real Options: A Practitioner's Guide." Norton.

[6] Cox, James C. and Ronald L. Oaxaca., 2000, "Good News and Bad News: Search from Unknown Wage Offer Distributions," *Experimental Economics*. 2: 197-225.

[7] Dixit, A. and Pindyck, A., 1994, "Investment Under Uncertainty." Princeton: Princeton University Press.

[8] Friedman, D. and Rust, J., 1993, "The Double Auction Market: Institutions, Theories and Evidence", Santa Fe Institute Proceedings Volume XIV, Addison-Wesley, 1993.

[9] Gong, B. and Ramachandran, V., 2007, "Payment Dominance in Sequential Search.", manuscript.

[10] Henry C., "Investment Decisions under Uncertainty: the Irreversibility Effect.", 1974, American Economic Review, 64(6), 1006-1012.

[11] Hirshleifer, J., Glazer A., and Hirshleifer D., 2005, "Price Theory and Applications", 7th ed. Cambridge.

[12] Kaplan, E., and Meier, P., 1958. "Nonparametric estimation from incomplete observations." *Journal of the American Statistical Association.* 53:457-481

[13] Kluger, B. and Wyatt, S., 2004, "Are Judgement Errors Reflected in Market Prices and Allocations? Experimental Evidence Based on the Monty Hall Problem" *Journal of Finance* 59: 969-997

[14] Kreps, D., 1990. "A Course in Microeconomic Theory." Princeton: Princeton University Press.

[15] MacDonald, R. and Siegel, D., 1986, "The Value of Waiting to Invest," *Quarterly Journal of Economics.* 101: 707-728.

[16] Moel, A. and Tufano, P., 2002, "When are Real Options Exercised? An Empirical Study of Mine Closings," *Review of Financial Studies*, 15: 35-64.

[17] Paddock, J., Siegel, D. and Smith, J., 1988, "Option Valuation of Claims on Real Assets: The Case of Offshore Petroleum Leases," *Quarterly Journal of Economics.*" 103: 479-508.

[18] Quigg, L., 1993, "Empirical Testing of Real Option-Pricing Models," *The Journal of Finance*, 48: 621-640.

[19] Robson, A., 2002, "Evolution and Human Nature." *Journal of Economic Perspectives.* 16: 89-106.

[20] Selten, R. and Buchta, J., 1998, "Experimental Sealed Bid First Price Auctions with Directly Observed Bid Functions." In D. Budescu, I. Erev, and R. Zwick (eds.), *Games and Human Behavior: Essays in Honor of Amnon Rapoport.* Mahwak, NJ: Erlaaum Ass.

A Mathematical Details

go here

B Supplementary Data Analysis

We cannot reject the proportional hazards assumption for these covariates. Again, move this footnote to the appendix and explain...

We observe significant ordering effects in monopoly blocks: Investment behavior changes markedly after the Competition block under each parameter set. The effect is to dampen the influence the influence of optimal option premium on investment. Because of this contamination we focus attention on the first monopoly block when reporting on monopoly behavior. We relegate details of this contamination, which is of second order importance to the paper, to Appendix X...so do it here? None of the statistical results reported in this section depend on this restriction.

C Econometric Details

put in material on PL and log-rank

revise the following on WLS:

We can then include all of these covariates in the following linear model

$$\mu_t - \mu_{t-1} = \alpha Lose_{t-1}(v_{t-1}^w - c_{t-1}) + \beta Win_{t-1}(v_{t-1}^w - v_{t-1}^s) + \kappa NoChance_{t-1} \\ + \delta Random_{t-1}(v_{t-1}^- - c_{t-1}) + \phi(c_t - c_{t-1}) + \epsilon_{t-1}$$

where ϵ is distributed $N(0, \sigma^2)$. An obvious problem with estimating such a model is that in many periods, we do not observe a subject's investment attempt and therefore fail to observe μ. In particular this happens in cases where *Random* or $NoChance$ are equal to one. We deal with this by instead estimating

$$\hat{\mu}_t - \hat{\mu}_{t-1} = \alpha Lose_{t-1}(v_{t-1}^w - c_{t-1}) + \beta Win_{t-1}(v_{t-1}^w - v_{t-1}^s) + \kappa NoChance_{t-1} \\ + \delta Random_{t-1}(v_{t-1}^- - c_{t-1}) + \phi(c_t - c_{t-1}) + \epsilon_{t-1}$$

where $\hat{\mu}$ is the observed μ in periods in which it is observed and is a latent variable in those periods in which it is not observed.

Consider a period t in which μ is observed and suppose that period N_t period precede it in which μ is not observed. Then we can rewrite our latent model entirely in terms of observables:

$$\begin{aligned}\mu_t - \mu_{t-N_t-1} = {} & \alpha Lose_{t-N_t-1}(v^w_{t-N_t-1} - c_{t-N_t-1}) + \beta Win_{t-N_t-1}(v^w_{t-N_t-1} - v^s_{t-N_t-1}) \\ & + \sum_{i=t-N_t}^{t-1} [\kappa NoChance_i + \delta Random_i(\bar{v}_i - c_i)] + \phi(c_t - c_{t-N_t-1}) + \sum_{k=t-N_t-1}^{t-1} \epsilon_k\end{aligned}$$

As long as we limit attention to this linear versions of the model, we can estimate such a function via standard weighted least squares. First, for each subject, we set μ_0 equal to the markup used in the subject's first observed investment attempt, dropping previous data. Second, we form the variable $\mu_t - \mu_{t-N_t-1}$ and estimate

$$\begin{aligned}\mu_t - \mu_{t-N_t-1} = {} & \alpha Lose_{t-N_t-1}(v^w_{t-N_t-1} - c_{t-N_t-1}) + \beta Win_{t-N_t-1}(v^w_{t-N_t-1} - v^s_{t-N_t-1}) \\ & + \sum_{i=t-N_t}^{t-1} [\kappa NoChance_i + \delta Random_i(\bar{v}_i - c_i)] + \phi(c_t - c_{t-N_t-1}) + \psi_t\end{aligned}$$

where ψ_t is distributed $N(0, \sigma^2 N_t)$.

D Instructions to Subjects