Time Varying Corporate Capital Stocks, the Cross Section and Intertemporal Variation in Stock Returns

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Abstract

We investigate an equilibrium model of an economy in which firm managers seek to maximize their individual firm's value through the costly adjustment of their capital stock in response to economic shocks. These economic shocks impact both the number of capital units each firm has and how profitable each unit is. The ultimate value of these corporate assets is determined by risk-averse investors that trade in a competitive multiple security market. The firm responds to higher profitability of risky capital by increasing its capital stock. This is balanced by higher risk that must be borne by investors, and hence a higher risk premium. In addition, because capital adjustment is sluggish, negative supply shocks to the stock of capital are only partially mitigated by investment, leaving the economy with an undersupply of risky capital. This leads to an equilibrium association between investment and a lower-than-usual risk premium. Our model firms therefore exhibit expected returns that increase with profitability and decrease with investment. Overall, the model predicts a vector VAR(1) structure for the state variables determining the cross-section of expected returns, and is broadly consistent with stylized facts (e.g., the value premium, size premium, earnings momentum, and investment premium). In addition, we predict and find additional empirical support for profitability as a determinant of firms' expected returns.

Stock returns appear to display a number of long run cross sectional and intertemporal patterns. One of the most studied is probably the tendency for returns to increase in a firm's book-to-market ratio and decrease in its size (Fama and French (1992)). But there are others. For example, stock returns appear to fall following equity issues[1] and rise following repurchases.[2] This relationship has since be reconfirmed in both the U.S. (Fama and French (2007) and Pontiff and Woodgate (2008)) and international (McLean, Pontiff, and Watanabe (2008)) markets. However, and more importantly for this paper, Titman, Wei and Xie (TWX, 2004) show that the equity issue and repurchase findings are in fact tied to a firm's investments. Firms that invest today tend to have lower returns going forward and vice versa (also see Lyandres, Sun, and Zhang (2007) and Xing (2007)).[3] Another example is the 'earnings momentum' phenomenon, noted by Bernard and Thomas (1989), in which firms with unusually high earnings post higher abnormal returns than those with unusually low earnings. The goal of this paper is to provide an explanation for why stock returns, corporate investment and profitability are related in a tractable general equilibrium framework and to generate a number of new testable cross-sectional predictions.

In the model both firms and investors play an active role in the determination of equilibrium prices and thus expected returns. Firms dynamically create goods and services across a number of industries by employing industry specific capital that varies over time. One source of time-variation in an industry's capital comes from employing

[1] See Marsh (1982), Asquith and Mullins (1986), Mikkelson and Partch (1986), Jung, Kim and Stulz (1996), and Baker and Wurgler (2002).
[2] See Ikenberry, Lakonishok, and Vermaelen (1995).
[3] Lyandres, Sun, and Zhang (2007) find that an investment factor explains most of the new issues puzzle, reducing 75% to 80% of SEO and IPO underperformance. Motivated by the q-theory, Xing (2007) shows that an investment growth factor does about as well as the value factor, driving out the value effect.

individuals. Employees sell their human capital at a fair price to a firm, which in turn converts the human capital into corporate capital. Another source of variation comes through the creation or dismantling of capital stock through a firm-specific proprietary technology. It is assumed that if a firm adds or subtracts from its capital stock in this manner it is relatively less expensive to do so slowly, meaning that capital adjustment to shocks is sluggish in equilibrium and that investment in the proprietary technology yields a positive net present value (NPV). The demand side of the model comes from risk-averse investors that own shares in the industries and trade them in a competitive market.

By using an overlapping generations framework based on Spiegel (1998) and related to those in Watanabe (2008), and Biais, Bossaerts, and Spatt (2008) the model is simultaneously tractable and capable of capturing a great deal of heterogeneity in the level, volatility, persistence, and correlations of cash flows generated by the panel of model firms. Another nice feature of the model is that the CAPM holds. However, while it holds period-by-period the model is not static. The return an investor can expect to earn by investing in an industry varies over time with the industry's profitability and investment activity. In particular, the CAPM beta is a decreasing function of capital investment in positive NPV projects and an increasing function of profitability. This is because the firm responds to shocks that increase the profitability of risky capital by increasing its capital stock, and the increased risk must be borne by investors who require a higher risk premium. On the other hand, because capital adjustment is sluggish, negative supply shocks to the stock of capital are only partially mitigated by positive NPV investment, leaving the economy with an undersupply of risky capital. When controlling for investment due to high profitability, this leads to an equilibrium association between investment and a lower-than-usual risk premium.

One can think of capital units in this setting somewhat more concretely by considering a firm such as Tyson Industries which is in the poultry business. In their case, a capital unit would be a chicken farm and the cash flow would be the profit per farm (or somewhat equivalently the price per pound of poultry produced). In the model and reality, the number of Tyson farms varies over time as their employees are able to build and improve them to a greater or lesser degree each period. At the same time the profits generated by each farm also varies with poultry and feed prices. If there are no adjustment costs to creating or dismantling farms, Tyson would respond to an increase in farm profitability by increasing the number of farms. This would be done instantaneously by all poultry producers until the price of capital equaled the cost of its creation. The now larger stock of risky farms borne by investors would be reflected in a higher risk premium. Thus shocks to profitability lead to higher expected returns.

At some point, the number of farms may be too low relative to a steady state (because once low profits have returned to their steady state or because some farms have been unexpectedly wiped out by disease), and the presence of capital adjustment costs ensures that this cannot be remedied instantly. The price of a farm will therefore be higher than usual despite the positive NPV investment taking place. The unusually low number of (risky) farms will signal an increase in risk-bearing capacity by the industry's investors, who will in turn command a lower-than-usual risk premium. Thus, sluggish though value-creating investment accompanies lower-than-usual expected returns. Eventually, investment in farms will move the industry back towards its steady state.

The relationships just described between profitability shocks, positive NPV investments, and expected returns reflect patterns identified in the literature (TWX as well as the prior literature on stock sales and repurchases, and Bernard and Thomas, 1989). That is, high stock returns are accompanied by an immediate increase in capital accumulation. Afterwards returns are below normal and capital accumulation in the industry tapers off. But, as in TWX the return phenomena are tied to changes in corporate capital levels and not financial issues or retirements *per se*.[4]

We provide empirical evidence that is consistent with the above story. Our model implies that the profitability of corporate capital and capital investment are the key variables to determine the cross-sectional variation in stock returns. We measure profitability by the ratio of earnings per unit capital to the cost of creating unit capital. We find that the zero investment portfolio that goes long high-profitability growth firms and short low- profitability growth firms earns a value-weighted average return of 0.81% per month. The risk-adjusted alpha from the standard four factor model is 0.92% per month. Both of these numbers are not only statistically significant (at the 1% level), but also economically significant.

This paper is not the first to theoretically examine the relationship between stock returns and both real and financial corporate capital adjustments. A number of authors have proposed behavioral explanations in which managers take advantage of overvalued shares to raise capital (Loughran, Ritter, and Rydqvist (1984), Ritter (1991), Loughran and Ritter (1995), Rajan and Servaes (1997), Pagano, Panetta, and Zingales (1998), Baker and Wurgler (2000) and Lowry (2003)). In contrast, recent models by Pastor and Veronesi (2005), Dittmar and Thakor (2007), and Li, Livdan, and Zhang (2009) offer

[4] If one associates any amount of costs with the issuance or repurchase of equity, then it stands to reason that equity issuance or repurchase is associated with positive NPV opportunities, from the point of view of management.

rational explanations linking observed capital expenditures and returns.

Like Pastor and Veronesi (2005), Dittmar and Thakor (2007), and Li, Livdan, and Zhang (2009) this paper also proposes a rational model that generates return series similar to what is seen in the data. In both Pastor and Veronesi and Li, Livdan, and Zhang market conditions change exogenously over time (meaning the pricing kernel changes accordingly) along various dimensions that lead to both time varying investment and subsequent returns. In Dittmar and Thakor a firm's managers and the investing public may not agree on the value associated with a new investment. When the divergence is large firms finance with debt, and when it is small with equity.

This paper contributes to the above line of work by seeking to explain the phenomena between investment and returns documented in TWX via a general equilibrium framework. That allows the model to examine not only time variation in returns, but betas, cross sectional patterns, and the relationship these all bear to variables like industry productivity. In both the Pastor and Veronesi (2005), and Li, Livdan, and Zhang (2009) paper market conditions are exogenous and firms react to them, here they are endogenous and influenced by the firms. This interplay allows the model to also make some predictions regarding how overall capital investment impacts the future trajectory of the economy. Also, whereas Dittmar and Thakor (2007) look at how heterogeneous beliefs influence returns in this article everyone has identical beliefs.

Other related models are those by Berk, Green and Naik (1999), Carlson, Fisher, and Giammarino (2004, 2006). These authors use real options models to examine how a firm's expected return will vary over time. Part of their focus is on the relationship between a firm's book-to-market and size that they generate, which look like those found in Fama and French (1992).[5] The firms in this paper have a much simpler investment problem than those in the articles cited above, yet they still generate similar book-to-market return patterns. Another difference is in the data needed to corroborate each model's predictions. Using commonly available data sources it is often difficult to know where and to what degree real option values are influencing a firm's current stock price. In the model developed here one only needs information like the firm's current capital and investment levels. While that does not make the model any more or less likely to be "right" it does make it easier to test and potentially refute. Finally, as both Berk, Green and Naik (1999) and Carlson, Fisher, and Giammarino (2004) acknowledge, their models are set up in a partial equilibrium framework with either the pricing kernel or the demand function exogenously given. In contrast, our model again is a general equilibrium model in which prices equilibrate supply and demand through market clearing.

Another related model can be found in Kogan (2004). He presents a two consumption good – two industry model with irreversible real investment. The model presented here has multiple industries, a single consumption good and reversible (though costly) investment. How sticky an investment is clearly depends on the time period and industry in question. Thus, which model may best describe particular phenomena like cross sectional returns, in particular industries or in general, is naturally an empirical question that is far beyond this paper's scope.

The paper is structured as follows. Section 1 presents the model. Section 2 contains the analysis, followed by some empirical evidence in Section 3. Section 4 concludes.

[5] Other partial equilibrium models that looks at real options and time varying risk are Sagi and Seasholes (2007) and Hackbarth and Morellec (2008). In both, the exercise of options on assets affects a firm's risk characteristics.

1. A Competitive Model with Capital Adjustments

1.1 Setting

There are K production factors which the paper will also refer to as industry sectors. Each production factor is used by a continuum of competitive all-equity, value maximizing, and price-taking firms with mass of unity. There is a single risk free asset that pays r per period and serves as the numeraire with a constant value of 1.[6] The production factors evolve over time via:

$$N_t = N_{t-1} + \eta_t + Y_t \tag{1}$$

where N_t equals the $K \times 1$ vector of production factors, t the time period. One can view η_t as representing the influence of human capital on the total supply of corporate capital. In the model people are born with a human capital endowment which in aggregate equals η_t. Individuals are fairly compensated for their human capital which, through their employment, is then converted into corporate capital having the impact shown in (1). From the perspective of investors η_t is a normally distributed random vector with mean zero and variance-covariance matrix Σ_η. The Y_t term is a $K \times 1$ vector of capital created by firms using a proprietary technology in the normal course of their business.[7]

In each period the production factors pay a $K \times 1$ dividend vector D_t that evolves via:

$$D_t = D_{t-1} + G\left(\bar{D} - D_{t-1}\right) - HN_t + \delta_t. \tag{2}$$

[6] As will be made clear later on the risk free asset is really just like the other K assets except that it is available for use by all industries.

[7] Other functional forms with various interpretations are clearly possible. For example, it is possible to change the assumption that employee capital contributions have a mean of zero by including a depreciation component to (1). In the long run capital stocks will then adjust so that depreciation offsets the average capital added by labor.

Here G and H are $K \times K$ matrices of constants, $\bar{D}$ is a $K \times 1$ vector of constants, and δ_t is a $K \times 1$ normally distributed random vector with zero mean and variance-covariance matrix Σ_δ.[8] Intuitively, the vectors $\bar{D}$ and N_t in (2) represent the impact on an asset's cash flows from demand and supply respectively. The $\bar{D}$ vector thus represents long run demand for the industry output. When h_{ij}, the ij^{th} element of H, is positive the output of industry j competes with that of industry i; if h_{ij} is negative then the output of industry j complements that of industry i. We will assume that G, $I - G$ and H are positive definite so that output is mean-reverting and the output of every industry competes with itself. The process in Eq. (2) is assumed to be stationary, meaning that while fads may temporarily boost prices in the model they eventually fade away. In terms of supply there is also implicitly an endogenous long run supply of industry assets (an equivalent $\bar{N}$). However, because this value is endogenous it is not included directly in the pricing equation.[9]

To help make the mathematical structure concrete consider again the poultry industry example, and for simplicity call it industry 1. In this case $d_{1,t}$ is the free cash flow per unit of capital in the industry (which is presumably tied to the market price of chicken). The random shock $\delta_{1,t}$ results from things like random diet fads that increase or reduce public demand for chicken. Fads however fade and the first element of G tells us the speed at which this typically happens. The other elements in the first row of G then relate the impact prices in other industries have on that speed. For example, a positive

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[8] We thank S. Viswanathan and Gur Huberman for suggesting that the model include N_t in the law of motion for D_t.

[9] This is without loss of generality as adding an $\bar{N}$ term simply alters the already existing constant in equation (2).

shock to fish prices might slow down how quickly a new diet emphasizing chicken will disappear.

Of course, changes in consumer demand are not the only thing that impact equilibrium prices. Production matters too and the matrix H captures this by mapping asset supplies to the cash flows generated by them. Again, looking at the poultry industry, the greater the number of farms (the higher $n_{1,t}$) the lower one expects the market price of chicken to be and thus the lower $d_{1,t}$ should be as well. But, other industries may also have an impact. If fish suppliers increase their productive capacity that may also lower poultry prices and thus the off diagonal elements of H may be non-zero as well.

1.2 Firms

Each firm's output comes from a single production factor.[10] Firms employ labor that creates real assets (or equivalently dismantles it or slows its rate of depreciation). In addition they also employ executives that determine if the firm should expend additional resources to take advantage of positive NPV opportunities through the firm's proprietary capital creation technology, thereby further adding to the capital created by the employee base. For both simplicity and to remain within the model's spirit it is assumed that, similarly to everything else, labor and executives are hired in a competitive market. This allows both labor and management to capture the value of their inputs to the firm. In particular, employees are fairly compensated for their contribution to the firm's capital base and managers captures all the rents from investing in positive NPV opportunities.

[10] In principle, firms can produce more than a single type of capital output. Assuming that the cost of building or liquidating capital is assessed at the firm level for each production factor separately, there is no loss of generality in considering firms that specialize only in a single type of output.

The managers of firm *fk* (i.e., firm *f* using factor *k*) can create value through the creation (or dismantling) of capital according to a quadratic cost schedule. This represents the proprietary capital creation technology of the firm. Taking the market price of capital as given, managers therefore face the following investment decision problem:

$$\max_{y_{fk,t}} y_{fk,t} p_{k,t} - c_{1k} y_{fk,t} - \frac{1}{2} c_{2k} y_{fk,t}^2, \qquad (3)$$

where $p_{k,t}$ is the period t market price of a unit of capital associated with the k^{th} production factor. In (3) $y_{k,t}$ equals the kth element of Y_t, and $y_{fk,t}$ the contribution of firm f to $y_{k,t}$. Thus, according to (3) $y_{fk,t}$ equals the units of capital above and beyond what the regular labor pool creates that management directs the firm to create.[12] The positive constants c_{1k} and c_{2k} represent the firm's linear and quadratic costs for adding a unit of capital to the k^{th} production factor. All firms in an industry are assumed to face the same costs c_{1k} and c_{2k}.[13] As mentioned earlier, managers are fairly compensated for their skill and therefore capture the value created from the maximization of Eq. (3).[14] If $c_{1k} > p_{t,k}$ then positive value is generated by the creation of *some* capital. The optimal amount is determined by the adjustment cost, c_{2k}. Thus, one can view the optimal choice of $y_{fk,t}$ as a measure of the positive NPV investment opportunities available to the firm.

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[12] There is no physical limit to the amount of new capital that can be deployed. Also, to maintain tractability new capital is financed only through the issuing (repurchase in the case of negative deployment) of equity. One could also allow for the use of riskless debt without any fundamental change to the model's results.

[13] Formally, firms have access to two production factors. One is exclusive to their industry. The other is the risk free asset. Other than being available to all industries the risk free asset is not particularly unique in any other way. One can place it within the same structure as the other K assets by setting its c_1, c_2 and the variance of its associated random shocks η and δ all to zero.

[14] As shown in the Appendix the model's competitive setting implies that maximizing (3) is equivalent to maximizing firm value as well. This particular formulation implicitly assumes managerial talent is in short supply allowing executives to capture the entire value of the rents they create. The Appendix discusses this assumption and why it simplifies the algebra but is not critical to the model's qualitative properties.

The c_{1k} and c_{2k} terms represent different aspects of the costs associated with creating productive capital. The c_{1k} parameter captures the base line cost of constructing a unit of production. Again consider a poultry producer like Tyson. For it c_{1k} equals the cost of building a new chicken farm. This ultimately depends on the price of raw materials like wood, wire, and trucks but not the market value of Tyson's own assets. Thus, the firm can potentially profit by building new farms when their market value exceeds their construction value and by selling them off when the reverse is true. The c_{2k} parameter captures the cost of increasing the speed with which assets are created or sold. Presumably, rushing the construction of a new chicken farm increases its ultimate cost but does allow the firm to generate cash flows from it earlier on. Naturally, whether a firm wishes to rush production of a new facility depends upon how much it expects to earn on it.

Differentiating (3) with respect to $y_{fk,t}$ (recalling that the firms take the price vector as given) and then solving for $y_{fk,t}$ yields for each production factor the capital issuance directed by management. Integrating over the set of firms and recalling that the total mass is set to unity then yields for each industry a capital addition of:

$$y_{k,t} = \left(p_{k,t} - c_{1k}\right) / c_{2k}, \tag{4}$$

where $y_{k,t} = \int y_{fk,t}\, df$ is the total amount of new capital deployed in factor k. For reference, let $N_{k,t} = \int n_{fk,t}\, df$ and $\eta_{k,t} = \int \eta_{fk,t}\, df$. Writing equation (4) in vector form:

$$Y_t = C_{2D}^{-1}(P_t - C_1), \tag{5}$$

where C_1 is the vector of linear costs with the k^{th} element c_{1k}, and C_{2D} is a $K \times K$ matrix with the k^{th} diagonal element equal to c_{2k} and zeros elsewhere thus:

$$C_{2D} = \begin{bmatrix} c_{21} & & & 0 \\ & c_{22} & & \\ & & \ddots & \\ 0 & & & c_{2K} \end{bmatrix}. \tag{6}$$

Finally, we note that the value created, per unit of capital, through positive NPV investment is $\frac{1}{2}C_{2D}Y_t$. This quantity plays a key role in the analysis of the equilibrium properties of the model.

1.3 Population

Investors, like firms, take prices as given. A continuum of investors with unit mass is born in period *t*, consume and then die in period *t*+1.[15] Investors have negative exponential utility functions with risk aversion parameter θ. Each investor begins life with an endowment of human capital, and immediately sells their human capital to firms (who subsequently convert it to corporate capital) and buys or sells securities to fund their end-of-period consumption.

Let $X_{i,t}$ represent the $K\times 1$ portfolio of share holdings of investor *i* in period *t*. Each share is assumed to represent one unit of a production factor. Let $w_{i,t}$ be the wealth with which investor *i* is born at date *t*. The assumption that people are born only with human capital implies $w_{i,t}$ equals the market value of that capital. Furthermore, because investors have negative exponential utility functions and all of the random variables are normally distributed the initial allocation of human capital does not impact the model's equilibrium results. Thus, all that is needed to proceed is knowledge that in the aggregate

[15] The assumption that investors live for one period of time is made only for tractability. There is no *a priori* reason to believe that this has a qualitative impact on how to calibrate the model to data. "Units of time" are determined by the speed at which capital can be accumulated and are not really dependent on the length of an agent's "life" (any more than the CAPM depends on investors living for one period).

the incoming human capital equals η_t and that those with skills associated with industry k will earn $\eta_{k,t} p_{k,t}$.

Based on the above discussion and letting $R\equiv 1+r$, an investor's period t+1 consumption equals:

$$X'_{i,t}\left(P_{t+1}+D_{t+1}-RP_t\right)+Rw_{i,t} \tag{7}$$

because it is assumed that he or she sells the portfolio prior to death. Recalling that the random vectors are normally distributed and investors have negative exponential utilities, investors maximize their expected utility by solving the following mean-variance problem:

$$\max_{X_{i,t}} E_t\left[X'_{i,t}\left(P_{t+1}+D_{t+1}-RP_t\right)+Rw_{i,t}\right]-\frac{\theta}{2}\operatorname{var}_t\left[X'_{i,t}\left(P_{t+1}+D_{t+1}-RP_t\right)+Rw_{i,t}\right]. \tag{8}$$

This reduces to,

$$\theta \operatorname{var}_t\left[Q_{t+1}\right]X_{i,t}=E_t\left[Q_{t+1}\right], \tag{9}$$

where

$$Q_{t+1}=P_{t+1}+D_{t+1}-RP_t \tag{10}$$

is the excess payoff vector from a unit position in each type of capital, and $\operatorname{var}_t\left[Q_{t+1}\right]$ is its variance-covariance matrix. Integrating over the continuum of investors and setting the market clearing condition $N_t=\int X_{i,t}\,di$ yields,

$$\theta \operatorname{var}_t\left[Q_{t+1}\right]N_t=E_t\left[Q_{t+1}\right]. \tag{11}$$

1.4 Equilibrium

Investors conjecture that prices are determined via the following formula:

$$P_t = A_0 + A_1 N_t + A_2 D_t \tag{12}$$

where A_0 is a $K\times1$ vector, while A_1 and A_2 are $K\times K$ matrices. At this point one can set up the set of simultaneous equations needed to derive the equilibrium parameters. However, the system is sufficiently complex that it yields few insights on its own and thus its derivation is left to Section 6.2 in the Appendix. More importantly, for much of the analysis that follows it turns out that, even without explicit solutions for the model's endogenous parameters, it is possible to say quite a bit about prices and returns.

In the rest of the paper, we will restrict our attention to equilibria in which A_1 is negative definite and A_2.[16] We can establish that this is the case in various instances.

Proposition 1: In each of the following limits an equilibrium exists in which A_2 is a positive definite matrix, while A_1 is a negative definite matrix.

1. $C_{2D}^{-1} \to 0$ for $\| H \|$ sufficiently small.
2. $C_{2D} \to 0$ for $\| H \|$ sufficiently small.
3. Σ_δ, Σ_η, G approach diagonal matrices and $\| H \|$ is sufficiently small.
4. $\theta\Sigma_\delta \to 0$, $(rI + G)^{-1} H$ is positive definite, and $\| H \|$ is sufficiently small.
5. $\theta\Sigma_\delta \to \infty$.

Proof. See the Appendix for the proof of this and all other propositions.

[16] Requiring A_1 to be negative definite is equivalent to requiring that $(P_t - E[P_t])' \cdot (N_t - E[N_t]) < 0$ whenever $D_t = E[D_t]$ and $N_t \neq E[N_t]$; i.e., if payoffs are at their steady state, prices and capital stock always move in opposite directions. Requiring A_2 to be positive definite is equivalent to requiring that $(P_t - E[P_t])' \cdot (D_t - E[D_t]) > 0$ whenever $N_t = E[N_t]$ and $D_t \neq E[D_t]$; i.e., if capital stocks are at their steady state, prices and payouts always move in the same direction.

It is straightforward to confirm that the equilibria with finite A_1 and $H = 0$ converge to those of Spiegel (1998) as $C_{2D}^{-1} \to 0$.

2. Analysis

2.1 Prices

Updating the time subscripts in (12) to t+1 and then plugging equations (1), (2) and (5) into equation (12), allows us to solve for P_{t+1} in terms of the parameter values known at time t and the unknown t+1 shocks:

$$\begin{aligned} P_{t+1} &= \left(I - \left(A_1 - A_2 H\right) C_{2D}^{-1}\right)^{-1} \times \\ &\left\{A_0 + \left(A_1 - A_2 H\right)\left(N_t + \eta_{t+1} - C_{2D}^{-1} C_1\right) + A_2\left[D_t + G\left(\bar{D} - D_t\right) + \delta_{t+1}\right]\right\}. \end{aligned} \tag{13}$$

Letting $F \equiv I - (A_1 - A_2 H) C_{2D}^{-1}$, and again using (1), (2) and (5) after a bit of manipulation, one arrives at the following Proposition.

Proposition 2: P_t and $\widehat{D}_t \equiv G(D_t + G^{-1} H N_t - \bar{D})$ follow the vector VAR(1) process,

$$\begin{aligned} P_{t+1} - C_1 &= F^{-1}\left\{(P_t - C_1) - A_2 \widehat{D}_t + \left(I - F\right) C_{2D} \eta_{t+1} + A_2 \delta_{t+1}\right\}, \\ \widehat{D}_{t+1} &= (I - G)\left\{\left[I - H C_{2D}^{-1} F^{-1} A_2\right] \widehat{D}_t + H C_{2D}^{-1} F^{-1} (P_t - C_1)\right\} \\ &+ (I - G)\left[H C_{2D}^{-1} F^{-1}\left(I - F\right) C_{2D} + H\right] \eta_{t+1} + \left[G + (I - G) H C_{2D}^{-1} F^{-1} A_2\right] \delta_{t+1}. \end{aligned} \tag{14}$$

As will be seen the VAR(1) process associated with prices will continue to propagate itself throughout the analysis. Indeed, this feature of the model will produce many of its empirical predictions. If the diagonal elements of $I - F$ are negative, a positive supply (η) shock to the k^{th} industry will negatively impact $p_{k,t}$. Likewise, if as assumed earlier, A_2 is negative definite then a positive payout (δ) shock to the k^{th} industry will positively impact $p_{k,t}$.[17]

Eq. (14) allows one to write the price as a function of past supply and payoff shocks as:

$$P_t = C_1 + \sum_{s=0}^{\infty} F^{-s-1}\left[\left(I-F\right)C_{2D}\eta_{t-s} + A_2\left(\delta_{t-s} - \widehat{D}_{t-s-1}\right)\right] \tag{15}$$

implying that the impulse response τ periods after a time t η-supply shock is given by $F^{-\tau-1}(I-F)C_{2D}\eta_t$. Similarly, the impulse response τ periods after a time t payout change is given by $F^{-\tau-1}A_2\left(\delta_{t-s} - \widehat{D}_{t-s-1}\right)$. As long as $\| F^{-1} \| < 1$, the effect of past shocks on prices eventually decays.[18]

Equation (15) provides a number of empirical predictions. At time 0 suppose a shock creates a large positive price move across stocks. Equation (15) suggests that following price series will decline. Note, this does not mean returns are negative as investors continue to receive a cash flow stream from the assets. When an industry capital unit fetches a value above its long run equilibrium value, firms in that industry increase their holdings of it (see equation (5)). Thus, if a shock generates a large price increase, that will in turn generate new investment by firms in the industry. This will be followed by lower capital prices in the industry, and reduced or even negative investment. The process continues on like this until a steady state is reached.

[17] It is possible to show that $I - F$ satisfies this property in the limits considered in Proposition 1.

[18] For a matrix X, we define $\| X \|$ to be the absolute value of the largest eigenvalue of X. As $\| X \|$ approaches zero, the matrix X approaches the zero matrix. It is possible to show that $\| F^{-1} \| < 1$ in cases 2,3, and 5 of Proposition 1 ($\| F^{-1} \| \to 1$ in the other two cases).

2.2 The Steady State

The economy is defined to be in a steady state in period t if firms do not actively seek to change their capital stock and if the expected change in the payout per unit of capital is expected to remain unchanged. This is a useful base case as it yields the model's predictions regarding unconditional moments, which could be compared to those in the data. From the steady state benchmark, one can see how various shocks to the system will impact estimated returns, risk factors and other financial and economic variables of interest.

Assuming the equilibrium state variables are mean-stationary, a steady state corresponds to a situation when the state variables coincide with their long-term means. Taking the unconditional means of the equations in equation (14) yields $E[P_t] = C_1$ and $E[\widehat{D}_t] = 0 = E[D_t] + G^{-1}HE[N_t] - \overline{D}$. Hence, the long-term mean of P_t is C_1, the long-term of mean of Y_t is zero, while the long term mean of D_t is :

$$E[D_t] \equiv \breve{D} = \overline{D} - G^{-1}H\overline{N}, \tag{16}$$

where $\overline{N} \equiv E[N_t]$, and can be expressed in terms of the model endogenous parameters (i.e., A_0, A_1 and A_2) through Eq. (12).

The conditional expected return to an investor from holding a claim in one unit of corporate asset k at date t equals

$$E_t\left[r_{k,t+1}\right] = \frac{E_t\left[p_{k,t+1} + d_{k,t+1}\right] - p_{k,t}}{p_{k,t}}, \tag{17}$$

where $d_{k,t}$ represents the k'th element of the vector D_t. Employing the steady state condition that $p_{k,t} = c_{1k}$ and $d_{k,t} = \breve{d}_k$ from Eq. (16), along with Proposition 2, lead to the following:

Proposition 3: If the economy is in steady state at date t then

$$E_t\left[r_{k,t+1} \mid \text{steady state at } t\right] = r + \frac{\theta\{V\bar{N}\}_k}{c_{1k}} = \frac{\breve{d}_k}{c_{1k}} = \frac{\bar{d}_k - \sum_{j=1}^{K} z_{k,j}\bar{n}_j}{c_{1k}} \equiv ss_k, \qquad (18)$$

where $\bar{n}_j$ represents the j$^{\text{th}}$ element of $\bar{N}$, Z denotes the matrix $G^{-1}H$, with $z_{k,j}$ as its kj^{th} element, $V \equiv \text{var}_t\left[Q_{t+1}\right]$ is independent of time and explicitly given in Eq. (43), and where $\{\bullet\}_k$ corresponds to the k^{th} element in the vector expression inside the curly bracket

Proposition 3 implies that if a firm in industry k is in its steady state, the firm's stock returns will equal the long run ratio of the asset's per unit profitability to its unit creation cost. Further, the right side of (18) can (at least in principle) be calculated with commonly available data. For a firm one could use the long run average earnings ($\breve{d}_k$) divided by the book value of assets or similar measures of a firm's cash flow and costs of productive assets.

The expression $E_t\left[r_{k,t+1} \mid \text{steady state at } t\right] = \frac{\breve{d}_k}{c_{1k}}$ in Eq. (18) indicates a firm's steady state expected returns are independent of its cash flow risk or investor risk aversion. The reason for this is also provided by Eq. (18): In the model, the number of capital units deployed can adjust over time. This leads industries to add or subtract assets to the point where investors bear an optimal, or steady state, level of risk. Essentially, the stock of capital in equation (11) adjusts to the point where the benefits of bearing its risk is balanced with investors' risk aversion, and the steady state expected returns are equal to

$r+\frac{\theta\{V\bar{N}\}_k}{c_{1k}}$. This emphasizes one of the model's fundamental properties: The long run productive economy is infinitely pliable. Industries can expand or contract as necessary based on costs and prices until the two are in line. Because prices reflect cost as well as risk, supply must change until investors, according to their risk bearing capacity, set prices equal to the creation cost of capital. Thus, from the perspective of long run returns, asset values (which ultimately must equal their cost of production) and profitability incorporate all one needs to know when it comes to calculating steady state expected returns.

2.3 Returns

Generally, in a model with a downward sloping demand curve, a negative supply shock increases the current price. In the model, this is additionally associated with an observable change in capital investment in the same direction as the price change. The presence of adjustment costs ensure that shocks are not fully undone by capital investment. Intuitively, the net supply shortage will mean that investors can bear more risk than before, thereby leading to lower expected returns. Because net supply shortages are associated with positive capital investments, one expects to see a negative relation between expected stock returns and capital investment. This is consistent with what Lyandres, Sun, and Zhang (2007) find in the data.

To formally analyze the above scenario define industry *k*'s excess return as

$$r^e_{k,t+1}=\frac{q_{k,t+1}}{p_{k,t}}, \qquad (19)$$

where $q_{k,t+1}$ is the k^{th} element of Q_{t+1}. Throughout the rest of the paper assume that the price and supply of capital are positive.[19] The next proposition asserts that the expected excess return decreases with capital investment.

Proposition 4: Assume that $E[r^e_{k,t+1}] > 0$. In each of the limits described in Proposition 1, an equilibrium exists in which, for every k, industry k's expected excess returns decrease with $y_{k,t}$ and increase with $d_{k,t}$:

$$\frac{\partial E\,[r^e_{k,t+1}]}{\partial y_{k,t}} < 0 \text{ and } \frac{\partial E\,[r^e_{k,t+1}]}{\partial d_{k,t}} > 0. \tag{20}$$

The first inequality is consistent with empirical relationship described in TWX and the related literature on new stock issuances and repurchases. In our model, firms issue or retire securities to buy or sell capital in response to shocks to the real economy that also impact equity prices. Although it may seem unintuitive that supplying the economy with a risky asset will *reduce* the equilibrium risk premium, our point is that such investment is generally a partial response to contemporaneous economic shocks that lead to an increase in the risk bearing capacity of the economy (e.g., negative shocks to the stock of capital). If firms did not invest, the risk premium would be even higher.

The second inequality in (20) is consistent with Bernard and Thomas (1989) and the earnings momentum literature. In the model, firms issue more capital in response to its higher profitability, given the current level of capital stocks. This is balanced by the increased risk premium investors require for bearing more capital stock risk.

[19] While both the price and supply are normally distributed in our model, one can arbitrarily reduce the probability of their assuming negative values. The distribution of the ratio of two Normal distributions is called the Fieller distribution and its application is abundantly found in the statistics literature.

2.3.1 Capital Investment and CAPM Beta

Because the model's random variables are normally distributed and the stock market is assumed to be competitive and frictionless, the CAPM must hold period by period. To verify this, we rewrite the equilibrium condition in equation (11) as

$$E_t[Q_{t+1}] = \theta \operatorname{cov}_t(Q_{t+1}, Q_{M,t+1}), \tag{21}$$

where $Q_{M,t+1} \equiv Q'_{t+1}N_t$ is the excess payoff on the market portfolio. Pre-multiply by N_t' to obtain

$$E_t[Q_{M,t+1}] = \theta \operatorname{var}_t(Q_{M,t+1}). \tag{22}$$

Dividing these two expressions side by side and rearranging, we have

$$E_t[Q_{t+1}] = \frac{\operatorname{cov}_t(Q_{t+1}, Q_{M,t+1})}{\operatorname{var}_t(Q_{M,t+1})} E_t[Q_{M,t+1}]. \tag{23}$$

Define the excess market return as

$$r^e_{M,t+1} \equiv \frac{Q_{M,t+1}}{P_{M,t+1}} = \frac{Q_{M,t+1}}{P'_{t+1} \cdot N_t}, \tag{24}$$

and rewrite equation (23) in terms of excess returns:

$$E_t[r^e_{t+1}] = \frac{\operatorname{cov}_t(r^e_{t+1}, r^e_{M,t+1})}{\operatorname{var}_t(r^e_{M,t+1})} E_t[r^e_{M,t+1}] \equiv \beta_t E_t[r^e_{M,t+1}]. \tag{25}$$

Where β_t is a K-dimensional vector with components

$$\beta_{k,t} = \frac{\operatorname{cov}_t(r^e_{k,t+1}, r^e_{M,t+1})}{\operatorname{var}_t(r^e_{M,t+1})}. \tag{26}$$

The following result follows immediately:

Corollary to Proposition 4: Assume that $\beta_{k,t} > 0$. In each of the limits described in Proposition 1, an equilibrium exists in which, for every *k*, industry *k*'s expected excess returns decrease with $y_{k,t}$ and increase with $d_{k,t}$:

$$\frac{\partial E\,[\beta^{e}_{k,t+1}]}{\partial y_{k,t}} < 0 \text{ and } \frac{\partial E\,[\beta^{e}_{k,t+1}]}{\partial d_{k,t}} > 0. \tag{27}$$

2.3.2 The Book-to-Market Ratio

We interpret $\frac{p_{k,t}}{c_{k1}}$ to be the market-to-book ratio, or Tobin's *q*, of a firm in industry *k*. This is justified by considering that c_{k1} is the cost of replacing a unit of capital in industry *k* as economically as possible. The firm's total book value in this case would be $c_{k1}n_{k,t}$, while its market value is $p_{k,t}n_{k,t}$. In the long run, the steady state requirement that $p_{k,t} = c_{1k}$ thus implies that the long run book-to-market ratio and thus Tobin's *q* for an industry should roughly equal 1. Consistent with this, the firm makes positive NPV investments in capital whenever the market-to-book ratio exceeds 1. Because of the adjustment costs, the book-to-market ratio deviates from 1. For example, if the cash flow (d_k) to a particular type of capital goes up so will the market value of that asset. This will decrease the book-to-market ratio and induce capital accumulation by firms in the industry.

Because Tobin's *q* for industry *k* in our model is equal to $\frac{c_{k2}y_{k,t}}{c_{k1}} + 1$, Proposition 4 and its corollary imply a cross sectional relationship between the book-to-market ratio and expected returns. A shock that decreases the book-to-market ratio today should be followed by future capital accumulation and lower than average expected returns to shareholders. This will continue until the "growth" stock sees its market-to-book (or equivalently Tobin's q) return to 1. The reverse will be true for "value" stocks.

The above analysis provides a rationale for the value-versus-growth return relationship that is both complementary to and separate from that in either Berk, Green and Naik (1999) or Carlson, Fisher and Giammarino (2004). In the prior models the premium results from firms altering their value through the exercise or expiration of growth options. Here the relationship also comes from capital changes in the underlying firms. But the firms in the model presented here do not exercise an option that leads to the price change, but rather react to one by building new capital that actually undoes the price change.[20]

2.4 *Return Dynamics Close to the Steady State*

One can get a sense of the behavior of prices, and therefore expected returns, by examining the price evolution in Eq. (14) in terms of small deviations of the state variables from their steady-state values. This is the purpose of the next Proposition:

Proposition 5: To linear order in Y_t and $D_t - \breve{D}$, the expected return on the stock of a firm in industry k is given by:

$$E_t\left[r_{k,t+1}\right] \approx ss_k - \frac{1}{c_{1k}}\left\{\left[(ss_k - r)I - \theta V A_1^{-1}\right] C_{2D} Y_t\right\}_k + \frac{1}{c_{1k}}\left\{(-\theta V A_1^{-1} A_2)(D_t - \breve{D})\right\}_k \quad (28)$$

where ss_k is defined in Eq. (18), and $\{\bullet\}_k$ corresponds to the k^{th} element in the vector expression inside the curly brackets.

[20] It is worth noting that while there is considerable evidence for a value premium in stock returns there is some question as to whether or not it is concentrated primarily in securities shunned by institutional investors. See Houge and Loughran (2006) and Phalippou (2007) for evidence on this issue.

If steady state expected excess returns (i.e., $ss_k - r$) are positive and, as is true under the conditions in Proposition 1, $-VA_1^{-1}C_{2D}$ is positive definite, then expected returns should be *negatively* related to investment (i.e., Y_t). Likewise, if $-VA_1^{-1}A_2$ is positive definite, expected returns are *positively* related to unusual profits. Proposition 5 embodies the main message of the paper: The equilibrium cross-section of expected returns should depend on the cross-section of profitability and investment, both in the long run and dynamically.

In the model, as in practice, the long run cash flows an asset generates depends not only on long-run demand for the product ($\bar{d}_k$) but also on its supply and that of competing or complementary products ($\sum_{j=1}^{K} z_{k,j}\bar{n}_j$). Both of these are present in the steady state expected returns, ss_k. In addition, Eq. (28) provides guidance on how deviations from the long-run, or steady state, affect returns. An example, on the demand side, could be the coal industry which could be falling out of favor for environmental reasons. Currently, profits are higher than what the steady state profitability is predicted to be. In this case, equation (28) predicts that, going forward, expected returns will decline because of the expected drop in profits; this, however, could be somewhat offset to the extent that the coal industry shrinks through value-creating (positive NPV) divestment. Whichever effect dominates depends on specifics of the industry. The opposite effects would, presumably, be anticipated for those industries producing coal alternatives such as manufacturers of solar panels.

Eq. (28) also gives an indication of how the returns of one industry might be impacted by the investment and profitability of related industries. For instance, even if the investment and profitability of solar panel manufacturers does not change from one period to the next, their expected returns could change because of declining profits, and divestment in the coal industry. In other words, expected returns in one industry will be impacted by changes in the fundamentals of complementary or substitute industries.

The factors that (28) relates to investor returns may help explain some of the advice commonly given by financial pundits. Casual observation indicates that they encourage the purchase of shares in industries with increasing profits. Conversely, they recommend selling shares in industries that are declining because competing ones are on the rise. Assuming financial pundits do not incorporate risk (however defined) into their recommendations, then they may simply tout stocks that they believe will have the highest expected returns.[22] Based on this, the model implies that advisors in the financial press will push stocks where (28) says the return is likely to be high and discourage ownership of those where (28) indicates they will be low. Casual observation and a consideration of what (28) implies about cross sectional returns in rising and declining industries seems to provide support for this hypothesis. Obviously, a more rigorous analysis would be needed to say anything truly definitive about this issue and there is no pretence that such has been carried out here.

2.5 Why Profitability and Investment Affect Expected Returns

As far as investors are concerned, only the price and supply of risky capital matter for expected returns. To see this, use Eq. (11), to write

[22] Essentially, this assumption amounts to positing that analysts do not consider the stock's riskiness or correlation with other stocks when issuing recommendations. Indeed, some television shows track the returns from holding their expert's picks but (so far as this paper's authors know) none track the volatility of the resulting portfolio.

$$E_t\left[Q_{t+1}\right] = \theta \operatorname{var}_t\left[Q_{t+1}\right] N_t = \theta V\bar{N} + \theta V(N_t - \bar{N}),$$

and then divide the k^{th} element of this vector by $p_{k,t}$ to get the expected excess return:

$$E_t[r_{k,t+1}] = \frac{\left\{\theta V\bar{N} + \theta V\left(N_t - \bar{N}\right)\right\}_k}{p_{k,t}}. \tag{29}$$

To understand how and why profitability and investment affect expected returns, it is useful to consider an economy in its steady state at date-(t-1) that is subsequently perturbed by an exogenous shock. Using Eq. (1) one can rewrite Eq. (29) as

$$E_t[r_{k,t+1}] = \frac{\left\{\theta V\bar{N} + \theta V\left(y_t + \eta_t\right)\right\}_k}{p_{k,t}} \tag{30}$$

There are two shocks to consider: supply (η) shocks and payoff (δ) shocks. Consider, first, the impact on the steady state of a one-time positive supply shock to industry k. The oversupply of capital will prompt firms to divest (i.e., $y_{k,t} < 0$). If adjustment costs are non-zero, the divestment will not completely counter the supply shock (i.e., $y_t + \eta_t > 0$), leading to a contemporaneous price decline. Thus, according to Eq. (30), expected returns will be higher than in the steady state because the numerator is greater than in the steady state while the denominator is smaller than in the steady state. As time passes and the effect of the shock slowly diminishes (as divestment continues), the same effect prevails but to a lesser degree: The oversupply of capital stock is still only partially offset by divestment while prices are still below their steady state value, leading to higher expected returns than in the steady state. The reverse takes place in response to a negative supply shock.

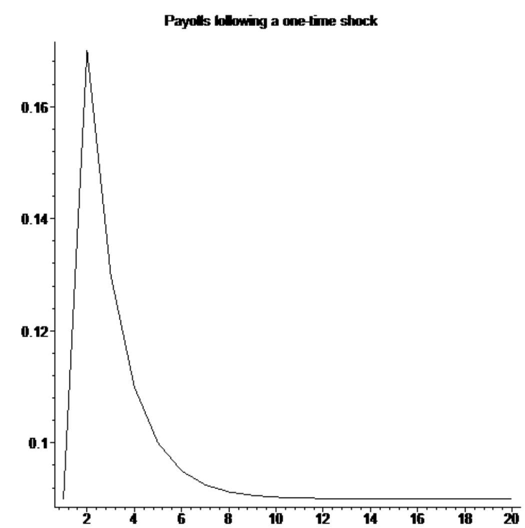
Summarizing, the presence of adjustment costs prevents firms from fully compensating for positive (negative) supply shocks via divestment (investment), forcing investors to bear more risk and leading to a higher (lower) risk premium. Thus, at least in reaction to supply shocks, divestment over time is unambiguously associated with a higher risk premium while investment is associated with a lower risk premium.

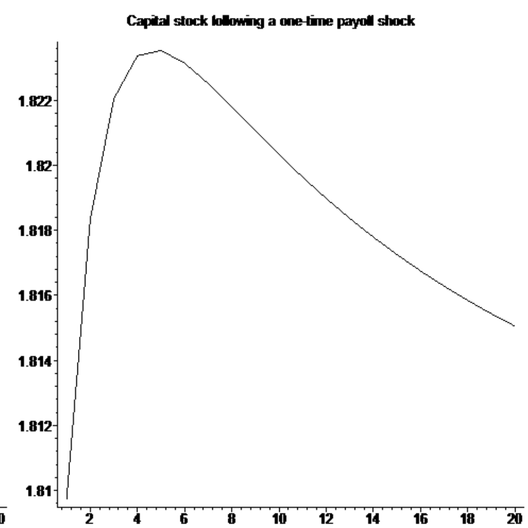
Consider now the impact on the steady state of a one-time positive *payoff* shock to industry k, turning off the η shocks so as to isolate the impact of a payoff shock on returns. Such a one-time shock makes owning capital more profitable and will therefore prompt firms to invest (i.e., $y_{k,t} > 0$). In this case, Eq. (30) does not unambiguously characterize the impact on expected returns.[23] The higher price of capital (in the denominator) will tend to reduce expected returns while the increase in capital stock will force investors to bear more risk and tend to increase expected returns. One can recast this tradeoff, however, as one between the response of capital stock to higher profitability (which is positively related to $D_t - \breve{D}$) and the negative response of expected returns to an increase in prices (which is negatively related to Y_t), just as in Eq. (28). This emphasizes the point that, in general, profitability and investment are not independent. In particular, when one assesses the sensitivity of expected returns to one of profitability or investments, one should control for the other (e.g., see Proposition 4, where the sensitivities are calculated as *partial* derivatives).

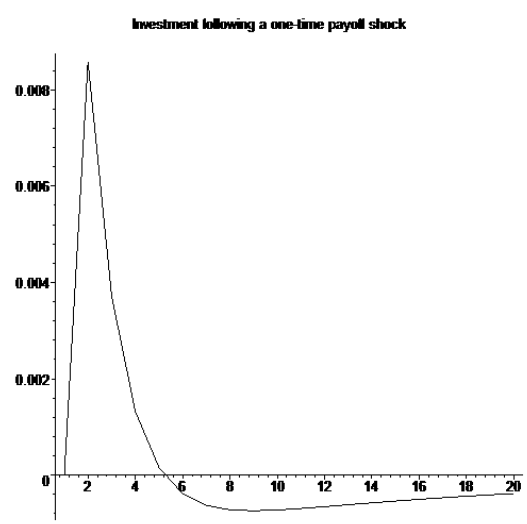
As time passes, profitability will revert to its long-term mean. At that point, the industry will have to slowly divest the extra capacity built to take advantage of the unusually high profits. Thus, the initial positive shock to payoffs eventually translates into an effective positive supply shock (once profits revert) with the same impact on

[23] For example, if C_{2D} is very small, prices adjust very quickly to shocks, and nearly all of the effect on expected returns is in the numerator of (30). On the other hand, if C_{2D} is very large, capital stocks do not change significantly and nearly all of the effect on expected returns is in the denominator of (30).

returns identified earlier for supply shocks. As an illustration, we plot below the impact on payoffs, capital stock and investment of a one-time date-2 shock, assuming the economy is in a steady state, and using the parameters in Section 2.7. In the illustration, the initial increase in capital stocks and investment (i.e., price) can have an ambiguous impact on expected returns. However, it is evident that once the rationale for having more capital stock diminishes with declining profitability (after about five years in the figures), the long-term impact of the payoff shock is akin to a supply shock.







Summarizing, investments are made for two reasons in our model: To mitigate supply shocks and to take advantage of increasing profitability. The profitability motive leads to a higher supply of risky capital and, therefore, a higher-than-usual risk premium or expected returns. When one controls for the profitability motive, investment is driven by supply shocks, and the presence of adjustment costs means supply shocks are only partially mitigated in equilibrium. In particular, positive investment takes place when there is an undersupply of risky capital, and is therefore followed by lower-than-usual risk premium and hence expected returns. This explains why investment and profitability enter into Eq. (28) with a negative and positive impact, respectively.

2.6 Other Limits of Interest

In this section, we provide explicit expressions for model quantities under certain limits, confirming the analysis in Section 2.4 and the assumptions that there exist equilibria exhibiting the properties we've explored in the previous sections.

2.6.1 Price behavior in the limits $H, \theta\Sigma_\delta \to 0$ and $\theta\Sigma_\delta \to \infty$

As the risk from δ becomes either very small or large relative to the population's risk aversion the equilibrium matrix equations approach relatively simple limits. This additional simplicity then makes it possible to generate more precise implications from the model.

Proposition 6: *As* $H, \theta\Sigma_\delta \to 0$*, an equilibrium exists for which the price goes to*

$$P_t \overset{H,\theta\Sigma_\delta \to 0}{\rightarrow} \frac{1+r}{r}(rI+G)^{-1}G\bar{D}_t + (rI+G)^{-1}(I-G)D_t, \qquad (31)$$

while as $\theta\Sigma_\delta \to \infty$*, the equilibrium prices goes to*

$$P_t \overset{\theta\Sigma_\delta \to \infty}{\to} \frac{1}{1+r}(C_1 + G\bar{D}) - \frac{1}{1+r}\theta\Sigma_\delta\left[\Sigma_\delta^{-1}C_{2D}\Sigma_\eta C_{2D} + I\right]N_t + \frac{1}{1+r}(I-G)D_t. \qquad (32)$$

The limit $\theta\Sigma_\delta \to 0$ corresponds to the case in which payoff risk or risk aversion is negligible. In this limit, because either investors are risk-neutral or payoffs are risk-free there should be no dependence on N_t because the profits from a unit of capital should simply be discounted by the risk-free rate. The limit $\theta\Sigma_\delta \to \infty$ corresponds to the opposite case. In both limits considered by the proposition $0 < \left\|F^{-1}\right\| < 1$ while both A_1 and $-A_2$ are negative definite. It is also possible to obtain results for the case $\theta\Sigma_\delta \to 0$ with H finite, but the expressions are far more complicated, requiring the solution of a quadratic matrix equation.

With a little bit of additional work equations (31) and (32) generate a number of comparative statics regarding how prices move in response to various state variables.

Proposition 7: *Assume that all matrices are diagonal. Then in the limits* $H, \theta\Sigma_\delta \to 0$ *and* $\theta\Sigma_\delta \to \infty$, $\partial p_{k,t} / \partial d_{k,t} > 0$ and $\partial p_{k,t} / \partial n_{k,t} < 0$. *Moreover,* $\lim_{H,\theta\Sigma_\delta \to 0} \frac{\partial p_{k,t}}{\partial d_{k,t}} < \lim_{\theta\Sigma_\delta \to \infty} \frac{\partial p_{k,t}}{\partial d_{k,t}}$ *and*

$$\lim_{H,\theta\Sigma_\delta \to 0} \left|\frac{\partial p_{k,t}}{\partial n_{k,t}}\right| < \lim_{\theta\Sigma_\delta \to \infty} \left|\frac{\partial p_{k,t}}{\partial n_{k,t}}\right| .$$

The comparative statics relate the laws of supply and demand with respect to real assets to stock prices. The result that $\partial p_{k,t} / \partial d_{k,t} > 0$ states that the value of an asset increases if the cash flow it generates increases. On the other hand, $\partial p_{k,t} / \partial n_{k,t} < 0$ implies that if the supply of an asset goes up then its price has to come down to clear the market. Returning to the Tyson example the first inequality states that if the price of poultry increases so does the value of a poultry farm. In contrast, the second inequality shows that if there is an overall increase in the number of such farms in the economy then the value of each farm will decline. The third and fourth inequalities show that an economy with a low cash flow risk (δ) shows less price sensitivity to cash flow and asset supplies shocks.

2.6.2 Profits and approximate expected returns in the limits $H, \theta\Sigma_\delta \to 0$ and $\theta\Sigma_\delta \to \infty$

After some manipulation, one has in the two limits that

$$\mathrm{E}_t\left[Q_{t+1}\right] \overset{\theta\Sigma_\delta \to \infty}{\to} \left\{\breve{D} - rC_1\right\} - RC_{2D}Y_t + (I-G)(D_t - \breve{D}), \tag{33}$$

and

$$\mathrm{E}_t\left[Q_{t+1}\right] \overset{H,\theta\Sigma_\delta \to 0}{\to} \left\{\breve{D} - rC_1\right\} - rC_{2D}Y_t + r(rI+G)^{-1}(I-G)(D_t - \breve{D}). \tag{34}$$

Each limit can be decomposed into a steady state term plus terms that depend on the deviations of the various state variables from their long-run means. Because *I-G* is positive definite, in both instances excess profits are positively related to capital payouts, D_t, and negatively related to capital issuance Y_t. Moreover, the sensitivities to these state variables sensitivities are more pronounced when cash flow risk (δ) or risk aversion is high. We note that the limit results in Eqs. (33) and (34) also apply to the dynamic properties of profit Sharpe ratios because the conditional variance-covariance matrix of Q_{t+1} is constant.

Assuming that G is diagonal, one obtains simple and informative expressions for the limiting behavior of expected returns. By writing $p_{k,t} = c_{2k} y_{k,t} + c_{1k}$, and dividing the excess profit of industry k by the price of a unit of industry k's capital, we get from equations (33) and (34) that the excess return on industry k's shares is

$$\mathrm{E}_t\left[r^e_{k,t+1}\right] \overset{\theta\Sigma_\delta \to \infty}{\to} \frac{\breve{d}_k - c_{2k} y_{k,t} + (1-g_k)(d_{k,t} - \breve{d}_k)}{c_{2k} y_{k,t} + c_{1k}} - r, \tag{35}$$

and

$$\mathrm{E}_t\left[r^e_{k,t+1}\right] \overset{H,\theta\Sigma_\delta \to 0}{\to} \frac{\breve{d}_k + \dfrac{r}{r+g_k}(1-g_k)(d_{k,t} - \breve{d}_k)}{c_{2k} y_{k,t} + c_{1k}} - r. \tag{36}$$

Expected excess returns in (36) do not approach zero because the amount of risky capital in the economy diverges in the limit. Denote the steady state expected returns ss_k as in Eq. . For small shocks away from the steady state, one can linearly approximate Eqs. (35) and (36) in the shocks as

$$\mathrm{E}_t\left[r^e_{k,t+1}\right] \overset{\theta\Sigma_\delta \to \infty}{\to} \approx \underbrace{ss_k - r}_{\text{steady state part}} - \left(ss_k + 1\right)\frac{c_{2k}}{c_{1k}} y_{t,k} + \frac{(1-g_k)}{c_{1k}}(d_{k,t} - \breve{d}_k), \tag{37}$$

and

$$\mathrm{E}_t\left[r^e_{k,t+1}\right] \overset{H,\theta\Sigma_\delta \to 0}{\to} \approx \underbrace{ss_k - r}_{\text{steady state part}} - ss_k \frac{c_{2k}}{c_{1k}} y_{t,k} + \frac{r}{r+g_k}\frac{(1-g_k)}{c_{1k}}(d_{k,t} - \breve{d}_k). \tag{38}$$

In each case, the deviation from steady state expected returns are negatively related to capital issuance, Y_t, and positively related to deviations of capital payout from the unconditional mean. As with the excess profits and Sharpe Ratios, the sensitivities are more pronounced when risk or risk aversion is high.

2.7 Cross-sectional returns – A Rough Calibration Exercise

The analytic results in the preceding sections provide a qualitative sense that the model can be consistent with a number of stylized facts in the literature. In order to illustrate this better we choose a set of parameters highlighting the cross-sectional effects. Our intention is not to perform a full scale calibration to industry data and cross-sectional moments at this stage (as is done for cross-sectional moments in Carlson, Fisher, and Giammarino, 2004).

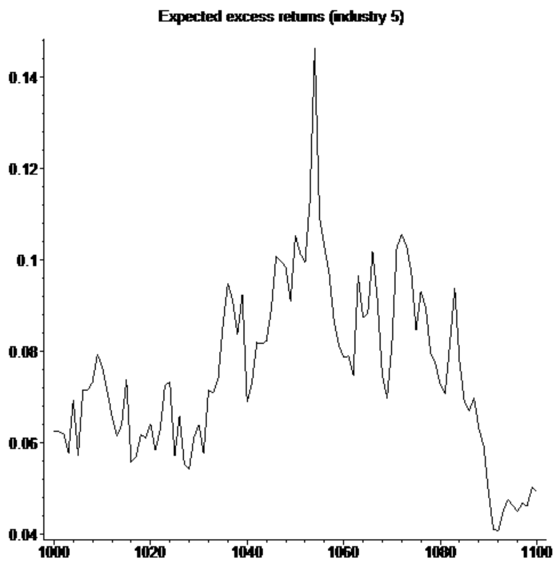
We consider the case of ten iid industries, where Σ_δ, Σ_η, G, and H are proportional to the identity matrix. We focus on ten industries so that we can construct the equivalent of cross-sectional deciles when calculating return moments. Each period corresponds to a year. Without loss of generality, we normalize the steady-state book value of one unit of capital to be 1. The risk free rate is chosen to be $R = 1.01$, consistent with the realized real rate of return over the past half century, while $\bar{d}$ is chosen to be 0.09 and $H = hI$ with $h = 0.005$. We set the volatility of payoffs and supply to be $\sigma_\delta = \sigma_\eta = 8\%$ and the rate of payoff mean-reversion is $g = 0.5$ (where $G = gI$). Thus the steady state excess rate of return is

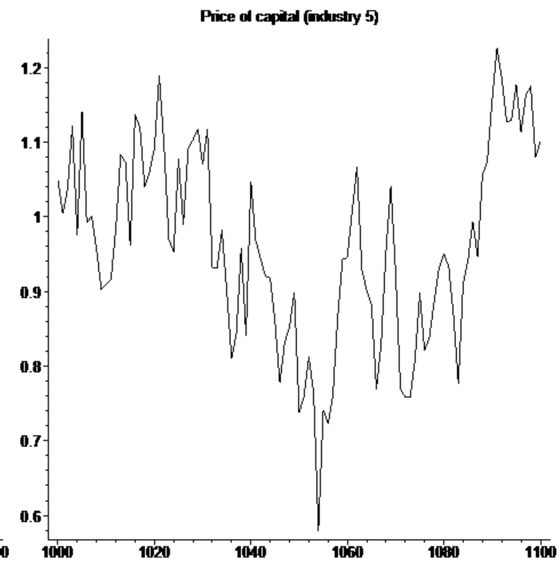
$$\frac{\breve{d}_k}{c_1} - r = \frac{\bar{d} - g^{-1} h \bar{n}_k}{c_1} - r = 0.08 - 0.01 \times \bar{n}_k, \tag{39}$$

with $\bar{n}$ determined endogenously. The remaining parameters are chosen to help arrive at 'reasonable' magnitudes for the stylized cross-sectional moments. Specifically, we set $\theta = 2$; then, in order to distinguish between the dispersion in market-to-book ratios (i.e., the dispersion in elements of P_t) and the dispersion in investment (i.e., the dispersion in elements of Y_t), we choose a heterogeneous set of adjustment costs. Elements of C_{2D} are symmetrically set according to the deciles of the normal distribution, $\mathcal{N}(8,4)$, with mean 8 and standard deviation 4. Explicitly, $c_{2,1}$ to $c_{2,4}$ are the first through fourth deciles of $\mathcal{N}(8,4)$, $c_{2,5}$ and $c_{2,6}$ are 8, and c_{27} to $c_{2,10}$ are the sixth through the last deciles of $\mathcal{N}(8,4)$. These parameters completely determine the model.

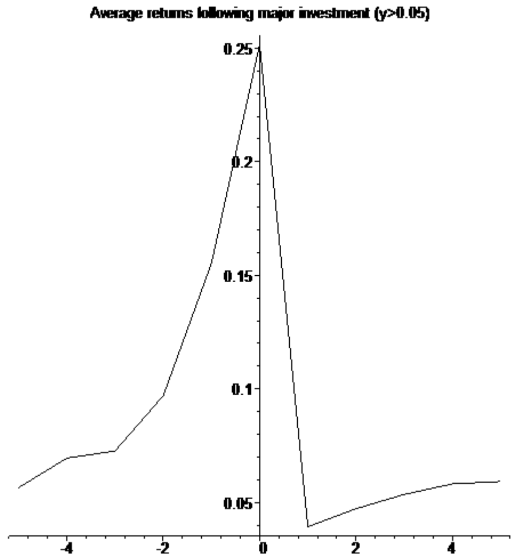
For each industry, the coefficient, $a_{1,k}$, solves a degree-five polynomial, which under our parameter specification has a unique negative real root (at -0.386 for the first industry and monotonically decreasing to -0.863 for the tenth industry); given that a negative real value for a_{1k} is the only sensible economic solution, this means that our particular parameter specification is not complicated by the presence of multiple equilibria. The remaining coefficients in the equation relating price to quantity and payoffs are: $a_{0,1}$ = 1.53, monotonically increasing to $a_{0,10}$ = 2.02, and $a_{2,1}$ = 0.876, monotonically increasing to $a_{2,10}$ = 0.924. Because the adjustment costs increase with the industry index, in the steady state industry size monotonically decreases from $\overline{n}_1$=1.55 to $\overline{n}_{10}$=1.27. The annual steady state expected excess returns vary between 6.45% for industry 1 and 6.73% for industry 10. The standard deviation of the price of a unit of capital varies from 0.144 to 0.163, thus the unconditional probability of a negative price realization is less than one in 10^9 years. The rates of mean-reversion of the price are determined by $(F^{-1})_{11}$ = 0.88, monotonically increasing to $(F^{-1})_{11}$ = 0.94. Industries with greater adjustment costs have capital prices that exhibit slower rates of mean reversion.

We simulate 10,000 years of the economy assuming that it is initially at the steady state. There was no instance in which the price fell below 0.29 or rose above 1.78. The average difference between the industry with the highest price per unit capital and that with the lowest price per unit capital is 0.48, with an average difference of about 6.4% in the amount of equity issued between the industry with the highest growth (i.e., positive y_t) and that with the lowest growth. Below is a plot of the price per unit capital in industry 5 after a 'burn-in' period of 1000 years; this is beside a plot of the corresponding expected excess returns for the same industry over the same period.





As explained earlier, when the price of capital is high expected returns tend to be low, and vice versa. The model also produces the familiar run-ups preceding major issuance events, and which are subsequently followed by declining returns. An instance of this is plotted below. The figure illustrates the average returns 5 years before and five years after a 'major investment' made by the leading investing industry (i.e., the event is said to take place whenever the leading industry makes an investment of y>5%). While the graph plots average realized returns (and not cumulative abnormal returns, or 'CARs'), it should be clear that the expect returns prior to the event are higher than the expected returns following the event, thus using a market model to adjust for risk will result in the usual observed patterns in CARs.



We calculate the market capital of each industry by multiplying its date *t* price per unit capital by the size of the industry. Consistent with the discussion in Section 2.3.2, we set the market-to-book ratio of an industry to be its price per unit capital. By sorting the industries with respect to size, market-to-book, investment, and payoffs, we can calculate the various asset pricing moments, reported below:

Portfolio	**Average Excess Returns**
SMB	1.4%
HML	6.3%
Low Investment	10.0%
High Investment	4.7%
High Payoffs	7.5%
Low Payoffs	6.4%

The SMB returns are the time-series average difference between the annual *expected* returns of the smallest industry (in market value) and the largest industry at date *t*. We use expected returns rather than realized returns to improve the power of the procedure (and because we can calculate these in our model). We only use the latter half of the simulated sample (using the first half makes a negligible difference given the number of significant digits we keep). The HML returns are the time-series average difference between the annual expected returns of the highest book-to-market industry and the lowest book-to-market industry at date *t*. Both the SMB and HML returns are consistent with stylized cross-sectional evidence in magnitude and sign. The book-to-market effect is discussed in Section 2.3.2. The size effect enters our model through two channels. Firstly, there is the channel discussed in Berk (1995): given two firms with identical cash flows, the smaller firm, tautologically, will be the one with the higher discount rate and therefore higher expected returns. Secondly, the steady-state returns in Eq. (39) negatively depend on $\overline{n}_k$, further contributing to a size effect.

The 'Low Investment' portfolio returns correspond to the time-series average of the lowest *y* industry expected returns at date *t*. The 'High Investment' portfolio returns are similarly calculated. Both are consistent with the observed issuance puzzle and Proposition 4. The difference between these two portfolio returns, about 5.3%, is distinct from the book-to-market (i.e., HML) premium. This is because heterogeneous adjustment costs allow for the highest market-to-book firm to be different from the highest investment firm. Finally, the payoff portfolios report a similar time-series average for the industry that happens to post the highest (resp. lowest) change in payoffs between dates *t*-1 and *t*. The results here are consistent with the earnings momentum phenomenon observed by Bernard and Thomas (1989): Firms that announce high (lower) earnings exhibit positive (negative) 'abnormal' returns relative to their pre-announcement risk-adjustment. This can be interpreted as a change in 'risk' subsequent to the announcement or that markets inadequately adjust for the impact of earnings announcements. Our model provides a risk-based explanation for this effect within an equilibrium framework: risk increases after large payoff news because of subsequent investment or divestment.

3. Empirical Evidence

The results in Section 2 and Proposition 3 imply that the book-to-market ratio (which is closely related to investments) and profitability are key variables to determining the cross-sectional variation in expected returns. We now examine this point empirically. Consistent with the model's implication, we will find that average returns increase with proxies of these two quantities.

3.1 Data and Methodology

We obtain accounting variables from the Compustat annual file. To measure profitability, we are guided by Eq. (28) to look for a proxy for D_t / c_{1k}. We compute this as the ratio of "Operating Income Before Depreciation" (Compustat Xpressfeed data item *OIBDP*, FTP data item 13) to contemporaneous "Property Plant and Equipment - Total (Gross)" (*PPEGT*, data item 7). This is consistent with our interpretation of c_{1k} as the steady state book value of capital. We denote this measure as *PROD*:

$$PROD_t = \frac{OIBDP_t}{PPEGT_t}. \tag{40}$$

The construction of the book-to-market ratio (*BM*) follows Fama and French (1993). Based on the firm characteristics at the end of fiscal year $t-1$, we form portfolios in June of calendar year t and measure value-weighted monthly returns from July through next June. The conservative six-month lag accounts for possible delay in the dissemination of accounting information and follows the usual practice. The monthly returns and variables necessary to compute market capitalization are from the Center for Research in Security Prices (CRSP), which are matched to the Compustat data by the CRSP-Compustat Merged Database. We use only ordinary common shares (CRSP Share Code 10 or 11) of firms in non-financial industries (one digit SIC code not equal to 6), because investment of financial firms may be very different in nature from that of non-financial firms. We use only NYSE firms (CRSP Exchange Code 1) to compute breakpoints for ranking, but include NYSE, AMEX, and NASDAQ firms (CRSP Exchange Code 1, 2, and 3) in portfolio formation. Our final sample runs from July 1968 through December 2006.

3.2 Result

3.2.1 One dimensional sort on *PROD*

Table 1 shows the characteristics, excess returns, and risk-adjusted alphas of decile portfolios sorted by *PROD*. The second column tells us that firms in the lowest *PROD* decile incur losses on average. The market capitalization (*SIZE*) tends to increase, and *BM* to decrease, with *PROD*, but the relations are not monotone. In fact, we will see variations in *BM* within a given *PROD* quintile, and vice versa, when portfolios are double sorted by these quantities in the next subsection. The table also indicates that there are relatively a large number of firms (*N*) in the top and bottom deciles; this implies that many NASDAQ firms fall in these two extreme *PROD* deciles, and that the point estimates of *SIZE* and *BM* may not properly represent the characteristics of firms in those deciles. To that extent, the excess return (*EXRET*) may not exhibit a linear relationship with *PROD*. This appears to be the case in the column for *EXRET*, from which one might incorrectly conclude that the underperformance of low *PROD* firms primarily comes from the lowest *PROD* decile only.

To account for the potential loadings on risk factors, we compute alphas from a time series regression of each excess portfolio return on the excess market return and the size, value, and momentum factors.[27] The estimated four-factor alpha (*ALPHA*) increases with *PROD* more monotonically, and tends to be negative for low profitability firms and positive for high profitability firms. While the zero-investment portfolio that goes long the highest profitability firms and short the lowest profitability firms earns a significant, albeit only moderate, average return of 0.36% per month, its risk-adjusted alpha is 0.56% per month and is significant at the 1% level. This demonstrates that, consistent with Proposition 4, firms with higher profitability earn higher expected returns and that this additional premium cannot be explained by existing risk factors. Another way to control

[27] The four factors are *MKTRF*, *SMB*, *HML*, and *MOM*, respectively, downloaded from Kenneth French's web site.

for existing priced factors is to further sort firms by the characteristics to which the risk factors are correlated. This is the subject of the next two subsections.

3.2.2 Two dimensional sort on *BM* and *PROD*

Table 2 presents the characteristics of 25 portfolios formed as the cross section of *PROD* and *BM* quintiles. Panel A indicates that average firms in the lowest *PROD* quintile again incur losses. Except for this quintile (and perhaps the highest-*PROD* fourth-largest *BM* portfolio), the level of profitability is controlled fairly well by the independent double sort. Panel B reports the average size in million dollars. Firms in the lowest *PROD* quintile tend to be small, especially for growth firms. If this has any implication on our result, the size effect will work against us; if high profitability firms tend to be large in size, we would expect them to earn low average returns, rather than high returns implied by Proposition 4. Panel C demonstrates that the independent double sort controls for the book-to-market ratio quite well, as there is little variation in *BM* along the columns. The number of stocks in Panel D assures us that each portfolio is well populated on average.

Panel E deserves attention. Excess return generally increases in *PROD* controlling for *BM*. The profitability spread, given by the return on a zero-investment portfolio that goes long the highest profitability firms and short the lowest profitability firms within a *BM* quintile, monotonically decreases with the level of *BM*. The long-short portfolio yields 0.81% per month among the growth firms, which is significant at the 1% level. On the other hand, the value spread is strongest among low profitability firms, yielding 1.05% per month. Interestingly, the value spread monotonically decreases with the level of *PROD*. The two numbers shown above are economically significant. A legitimate concern is that these spreads may partially reflect the reward for bearing known risks.

The four-factor alphas in Panel F control for this possibility. As anticipated, the value spread is significantly reduced after taking into account the loadings on the value and other factors. However, the profitability spread barely changes or even increases for growth stocks upon risk adjustment; the four-factor alpha of the zero-investment profitability portfolio is 0.92% among growth firms. This magnitude of alpha is not only statistically significant (at the 1% level), but also economically significant. The alpha decreases monotonically with *BM*. For concreteness, the next subsection further controls for size.

3.2.3 Three dimensional sort on Size, *BM* and *PROD*

Table 3 reports the characteristics of 27 portfolios formed as the cross section of *SIZE*, *BM*, and *PROD* terciles. For simplicity, we focus on the lowest and highest profitability terciles as we are interested in the profitability spread. Panel A shows the market capitalization of the nine *SIZE-BM* terciles at the lowest and highest profitability levels. Again, if there is any bias resulting from size, it will work against us because the highest profitability firms tend to be larger than lowest profitability firms, thereby reducing the profitability spread. Similarly, the book-to-market ratio in Panel B appears to be well controlled. Panel C confirms that the profitability spread is highest among small to mid growth firms, yielding 0.72% to 0.76% per month, both significant at 1%. These spreads barely change with risk adjustment; the four-factor alphas in Panel D for the corresponding portfolios are 0.68% and 0.61% per month, respectively. Indeed, the alpha for the largest growth portfolio is also significant at the 5% level, yielding 0.49% per month.

Overall, the empirical results presented in this section are consistent with Proposition 4, which says that high profitability firms should earn high returns. This profitability effect cannot be explained by existing risk factors.

4. Conclusion

Traditionally the asset pricing literature has taken the characteristics of the set of corporate assets as given when solving for the equilibrium returns demanded by investors. Recently a number of papers have begun to look at the problem when the characteristics of corporate assets change over time. Articles by Spiegel (1998), Watanabe (2008), Biais, Bossaerts, and Spatt (2008), Pastor and Veronesi (2005), Dittmar and Thakor (2007), Berk, Green and Naik (1999), and Carlson, Fisher, and Giammarino (2004, 2006) all fall into this category. This paper seeks to add to this literature a general equilibrium view of the problem. Both the pricing kernel and the equilibrium supply of productive capital are endogenous in our model. Corporate capital stocks are impacted by both random fluctuations and the investment decisions of value maximizing firms who seek to add and subtract from their capital base in response to market conditions. In turn, asset prices are determined period by period by risk averse investors via market clearing conditions. The end result is a tractable model that yields a number of empirical predictions, many of which are consistent with the data. Among these are the following:

- Stock returns should be positively correlated with a proxy for profitability of capital, such as the earnings yield on a firm's capital stock.

- Large returns (price moves) in one direction will be followed by a decaying series in the opposite direction.
- Capital expenditures will be negatively correlated with future returns.

Because the CAPM holds, period-by-period in the model, the above relationships regarding returns also hold for period-by-period betas. This, however, also implies that empirical models that do not allow time-varying betas will be incorrectly specified. In particular, the CAPM beta should be modeled as a decreasing function of capital investment.

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6. Appendix

6.1 Equivalence between Maximizing Executive Value and Firm Value

A firm *f*'s capital base at the end of period *t* in industry *k* is worth $n_{fk,t}p_{k,t}$. (Recall, $n_{fk,t}$ is the capital stock, and $p_{k,t}$ the market price of a capital unit.) To get to the $n_{fk,t}$ units of capital the firm ended with it in the current period it added $\eta_{fk,t}$ units from its employment of labor and $y_{fk,t}$ from management's decision to use additional capital resources. Thus, one has $n_{fk,t} = n_{fk,t-1} + \eta_{fk,t} + y_{fk,t}$. The additional capital units are not free and the model assumes that due to competitive markets labor captures the value of its input. For regular labor this yields a wage of $\eta_{fk,t}p_{k,t}$. Management's decisions add $y_{fk,t}p_{k,t} - c_{1k}y_{fk,t} - \frac{1}{2}c_{2k}y_{fk,t}^2$ to the firm's value. This then equals their compensation – call it $\xi_{fk,t}$. Thus, at the end of the period the firm's value equals:

$$\left(n_{fk,t-1} + \eta_{fk,t} + y_{fk,t}\right)p_{k,t} - c_{1k}y_{fk,t} - \frac{1}{2}c_{2k}y_{fk,t}^2 - \eta_{fk,t}p_{k,t} - \xi_{fk,t}. \tag{41}$$

Note the two $\eta_{fk,t}p_{k,t}$ terms cancel out. Also the $\xi_{fk,t}$ exactly offsets the $y_{fk,t}p_{k,t} - c_{1k}y_{fk,t} - \frac{1}{2}c_{2k}y_{fk,t}^2$ term. Thus, the end of period firm value equals $n_{fk,t-1}p_{k,t}$. This figure is not impacted by management's choice of $y_{fk,t}$ since each firm's perceives that its individual actions do not impact the overall equilibrium price per unit of capital (i.e. $p_{k,t}$ is a function of $\int n_{fk,t}df$). Therefore, maximizing managerial value equivalently maximizes the firm's market value, and similarly the overall value to its constituent stakeholders.

It is worth noting that one can drop the assumption that management earns the full or partial value of its input to the firm and not alter the model's qualitative properties. The important feature required by the model is that firm's seek to add assets when they have a high market value and subtract them when the value is low. Any modeling assumption that leads to this conclusion will yield similar results.

Here, at least, the assumption that labor and management capture the value of their input does keep the model's competitive markets setting consistent across all markets. Additionally, it also helps produce a closed form solution for the financial market's pricing function. In particular it makes the accounting particularly simple. An investor that owns X units of capital at the start of a period will own (in present value terms) the same number of units at the end of the period, since labor captures the value from changes to firm's capital base. Investors, by contrast, capture the value from fluctuations in the per unit capital values.

6.2 Derivation of the Equilibrium Conditions

Define $F \equiv \left(I - \left(A_1 - A_2 H\right) C_{2D}^{-1}\right)$. Use (10) and (14) to produce (after some algebraic manipulation):

$$\begin{aligned} E_t\left[Q_{t+1}\right] = &\left[\left(I - HC_{2D}^{-1}\right)F^{-1} - RI\right]P_t \\ &+ \left(I - HC_{2D}^{-1}\right)F^{-1}\left[A_2 G\left(\bar{D} - D_t\right) + (F - I)C_1 - A_2 H N_t\right] \\ &+ D_t + G\left(\bar{D} - D_t\right) - H\left(N_t - C_{2D}^{-1} C_1\right). \end{aligned} \tag{42}$$

Similarly,

$$
\begin{aligned}
\operatorname{var}_t\left[Q_{t+1}\right] &= \operatorname{var}_t\left[\left(I-HC_{2D}^{-1}\right)F^{-1}\left(\left(A_1-A_2H\right)\eta_{t+1}+A_2\delta_{t+1}\right)-H\eta_{t+1}+\delta_{t+1}\right] \\
&= \left\{\begin{matrix}\left[\left(I-HC_{2D}^{-1}\right)\left(F^{-1}-I\right)C_{2D}-H\right]\Sigma_\eta\left[\left(I-HC_{2D}^{-1}\right)\left(F^{-1}-I\right)C_{2D}-H\right]' \\ +\left[\left(I-HC_{2D}^{-1}\right)F^{-1}A_2+I\right]\Sigma_\delta\left[\left(I-HC_{2D}^{-1}\right)F^{-1}A_2+I\right]'\end{matrix}\right\} \\
&\equiv V.
\end{aligned}
\tag{43}
$$

To solve for the equilibrium values of the A's, replace $E_t\left[Q_{t+1}\right]$ and $\operatorname{var}_t\left[Q_{t+1}\right]$ in equation (11) with the corresponding terms in equations (42) and (43). The coefficients of N_t and D_t must vanish separately as well as those that do not multiply a time varying parameter. For the terms that multiply neither N_t or D_t this yields,

$$
\begin{aligned}
&\left[\left(I-HC_{2D}^{-1}\right)F^{-1}-RI\right]A_0+\left(I-HC_{2D}^{-1}\right)F^{-1}\left[A_2G\bar{D}+\left(F-I\right)C_1\right] \\
&\quad+G\bar{D}+HC_{2D}^{-1}C_1=0,
\end{aligned}
\tag{44}
$$

while for the terms multiplying N_t,

$$
\left[\left(I-HC_{2D}^{-1}\right)F^{-1}-RI\right]A_1-\left(I-HC_{2D}^{-1}\right)F^{-1}A_2H-H-\theta V=0,
\tag{45}
$$

and finally for the terms multiplying D_t,

$$
\left[\left(I-HC_{2D}^{-1}\right)F^{-1}-RI\right]A_2-\left(I-HC_{2D}^{-1}\right)F^{-1}A_2G+I-G=0.
\tag{46}
$$

The equilibrium values of A_0, A_1, and A_2 can now be found by solving (44), (45), and (46)
.

6.3 Proofs

Proposition 1: In each of the following limits an equilibrium exists in which A_2 is a positive definite matrix, while A_1 is a negative definite matrix.

1. $C_{2D}^{-1}\to 0$ for $\|H\|$ sufficiently small.

2. $C_{2D} \to 0$ for $\| H \|$ sufficiently small.

3. Σ_δ, Σ_η, G approach diagonal matrices and $\| H \|$ is sufficiently small.

4. $\theta\Sigma_\delta \to 0$, $(rI + G)^{-1} H$ is positive definite, and $\| H \|$ is sufficiently small.

5. $\theta\Sigma_\delta \to \infty$.

Proof. Taking the limit $C_{2D}^{-1} \to 0$, one can write equation (45) as

$$-rA_1 - A_2 H - H - \theta V = 0,, \tag{47}$$

where V is the covariance matrix of excess payoffs defined in equation (43). Because V is positive definite by construction, for $\| H \|$ sufficiently small A_1 is positive definite. Next, take the limit of (46) as $C_{2D}^{-1} \to 0$:

$$-rA_2 - A_2 G + I - G = 0. \tag{48}$$

Because G and $I - G$ are positive definite, so is A_2.

Now, taking the limit $C_{2D} \to 0$, it's straight forward to show that Eq. (45) has a solution $A_1 = -\frac{\theta}{R} V$, which is negative definite. Eq. (46) in this limit becomes $\frac{1}{R}(I - G) - \frac{1}{\theta} H V^{-1} A_2^2 = A_2$, which has a positive definite solution for $H = 0$, and therefore a positive definite solution whenever $\| H \|$ is sufficiently small.

To establish case 3, first set H to zero, assume Σ_δ, Σ_η, G are diagonal, and write equation (45) for the k^{th} diagonal element as

$$\left(\frac{1}{1 - a_{1,k} / c_{2,k}} - R \right) \frac{a_{1,k}}{c_{2,k}} = \theta \frac{V_{kk}}{c_{2,k}}.$$

For $a_{1,k} \in (-\infty, 0)$, $a_{2,k}$ is bounded as is V_{kk}, so a solution to the above equation exists with $a_{1,k} \in (-\infty, 0)$. This proves that A_1 is negative definite in this limit. In the same limit, given that $F_{kk} > 1$ and $0 < 1 - G_{kk} < R$, Eq. (46) implies A_2 is positive definite. Thus, long as Σ_δ, Σ_η, G are sufficiently close to being diagonal matrices and $\| H \|$ is sufficiently small, A_2 remains positive definite matrix, while A_1 remains negative definite.

To establish the fourth case, first consider the limit $H, \theta\Sigma_\delta \to 0$. We look for a solution for which $A_1 \to 0$. Thus, as $\theta\Sigma_\delta \to 0$ and for H small, we look for a solution to A_1 of the form $A_1 \sim \xi H$. To linear order in H Eq. (45) becomes $-r\xi H - A_2 H - H = 0$, yielding $A_1 = -\frac{1}{r}(A_2 + I)H$. Applying the limit to Eqs. (44) and (46) yields $A_2 = (rI + G)^{-1}(I - G)$, which is positive definite, and plugging back into the expression for A_1, $A_1 = -\frac{R}{r}(rI + G)^{-1}H.$

In the limit $\theta\Sigma_\delta \to \infty$, Eqs. (45) and (46) imply that A_1 diverges while A_2 approaches a finite quantity. In turn, this means that F^{-1} approaches 0. The consequent limiting behavior of Eqs. (45), and (46) can be summarized as

$$\begin{aligned} -RA_1 - H - \theta\left[C_{2D}\Sigma_\eta C_{2D} + \Sigma_\delta\right] = 0, \\ -RA_2 + I - G = 0. \end{aligned} \qquad (49)$$

A_2 in Eq. (49) is positive definite, while A_1 approaches a negative definite matrix as $\theta\Sigma_\delta \to \infty$.

■

Proposition 3: If the economy is in steady state at date t then

$$E_t\left[r_{k,t+1} \mid \text{steady state at } t\right] = r + \frac{\theta V\bar{N}}{c_{1k}} = \frac{\breve{d}_k}{c_{1k}} = \frac{\bar{d}_k - \sum_{j=1}^{K} z_{k,j}\bar{n}_j}{c_{1k}} \equiv ss_k,$$

where $\bar{n}_j$ represents the j[th] element of $\bar{N}$, Z denotes the matrix $G^{-1}H$, with $z_{k,j}$ as its kj^{th} element, and $V \equiv \text{var}_t\left[Q_{t+1}\right]$ is independent of time and explicitly given in Eq. (43).

Proof. The first equality follows from using Eq. (11) to write

$$E_t[P_{t+1} + D_{t+1}] = E_t\left[Q_{t+1}\right] + RP_t = \theta\,\text{var}_t\left[Q_{t+1}\right]N_t + RP_t = \theta V\bar{N} + RP_t + \theta V(N_t - \bar{N}), \quad (50)$$

dividing by P_t and setting the date-t variables to their steady state values. The second equality comes from using Eq. (14) in the expression for $E_t[P_{t+1} + D_{t+1}]$, dividing by P_t and setting the date-t variables to their steady state values.

■

Proposition 4: Assume that $E[r^e_{k,t+1}] > 0$. In each of the limits described in Proposition 1, an equilibrium exists in which, for every k, industry k's expected excess returns decrease with $y_{k,t}$ and increase with $d_{k,t}$:Proposition 3Proposition 3

$$\frac{\partial E\,[r^e_{k,t+1}]}{\partial y_{k,t}} < 0 \text{ and } \frac{\partial E\,[r^e_{k,t+1}]}{\partial d_{k,t}} > 0. \quad (20)$$

Proof. Using Eq. (11), write

$$E_t\left[Q_{t+1}\right] = \theta\,\text{var}_t\left[Q_{t+1}\right]N_t = \theta V\bar{N} + \theta V(N_t - \bar{N}) = \theta V\bar{N} + \theta V A_1^{-1}\left(C_{2D}Y_t + A_2(ND_t - \breve{D})\right). \,(51)$$

Dividing the k^{th} element of this vector by $p_{k,t} = c_{2k}y_{k,t} + c_{1k}$ to get the expected excess return gives,

$$E\,[r^e_{k,t+1}] = \frac{\left\{\theta V\bar{N} + \theta V A_1^{-1}\left(C_{2D}Y_t - A_2(D_t - \breve{D})\right)\right\}_k}{c_{2k}y_{k,t} + c_{1k}}. \quad (52)$$

Taking the derivative with respect to $y_{k,t}$ yields the correct sign with respect to the denominator term and yields $-c_{2k}\dfrac{\left\{\theta V\bar{N} + \theta V A_1^{-1}\left(C_{2D}Y_t - A_2(D_t - \breve{D})\right)\right\}_k}{(c_{2k}y_{k,t} + c_{1k})^2} + \dfrac{\left\{\theta V A_1^{-1}C_{2D}\right\}_{kk}}{c_{2k}y_{k,t} + c_{1k}}$.

The first term is negative as long as $E[r^e_{k,t+1}] > 0$. It should be clear from the proof of Proposition 1 that the second term is negative in cases 1-3 and 5. For case 4, set H to zero and note that Eq. (45) implies that VA_1^{-1} is negative definite, so this will remain true whenever $\|H\|$ is sufficiently small.

Taking the derivative of (52) with respect to $d_{k,t}$ yields $-\frac{\left\{\theta VA_1^{-1}A_2\right\}_{kk}}{c_{2k}y_{k,t}+c_{1k}}$. Following the same line of argument as with $\frac{\partial E\,[r^e_{k,t+1}]}{\partial y_{k,t}}$, it is straight forward to show that $VA_1^{-1}A_2$ is negative definite. ■

Proposition 5: To linear order in Y_t and $D_t - \breve{D}$, the expected return on the stock of a firm in industry k is given by:

$$E_t\left[r_{k,t+1}\right] \approx ss_k - \frac{1}{c_{1k}}\left\{\left[(ss_k - r)I - \theta VA_1^{-1}\right]C_{2D}Y_t\right\}_k + \frac{1}{c_{1k}}\left\{(-\theta VA_1^{-1}A_2)(D_t - \breve{D})\right\}_k \quad (53)$$

where $\{\bullet\}_k$ corresponds to the k^{th} element in the vector expression inside the curly brackets and $V \equiv \text{var}_t\left[Q_{t+1}\right]$ is independent of time and explicitly given in Eq. (43).

Proof. Replace $N_t - \bar{N}$ in Eq. (50) with $A_1^{-1}\left((P_t - C_1) - A_2(D_t - \breve{D})\right)$

$= A_1^{-1}\left(C_{2D}Y_t - A_2(D_t - \breve{D})\right)$. Dividing the k^{th} component of this by the price

$p_{k,t} = c_{1k} + c_{2k}y_{k,t}$, expanding the resulting expression to linear order in deviations from the steady state yields the desired result.

■

Proposition 6: *As* $H, \theta\Sigma_\delta \to 0$*, an equilibrium exists for which the price goes to*

$$P_t \overset{H,\theta\Sigma_\delta \to 0}{\to} \frac{1+r}{r}(rI+G)^{-1}G\bar{D} - \frac{R}{r}(rI+G)^{-1}HN_t + (rI+G)^{-1}(I-G)D_t, \qquad (31)$$

while as $\theta\Sigma_\delta \to \infty$*, the equilibrium prices goes to*

$$P_t \overset{\theta\Sigma_\delta \to \infty}{\to} \frac{1}{1+r}(C_1 + G\bar{D}) - \frac{1}{1+r}\theta\Sigma_\delta\left[\Sigma_\delta^{-1}C_{2D}\Sigma_\eta C_{2D} + I\right]N_t + \frac{1}{1+r}(I-G)D_t. \qquad (32)$$

Proof: First consider the limit $H, \theta\Sigma_\delta \to 0$. We look for a solution for which $A_1 \to 0$. Thus, as $\theta\Sigma_\delta \to 0$ and for H small, we look for a solution to A_1 of the form $A_1 \sim \xi H$. To linear order in H Eq. (45) becomes $-r\xi H - A_2 H - H = 0$, yielding $A_1 = -\frac{1}{r}(A_2 + I)H$. Applying the limit to Eqs. (44) and (46) yields $A_2 = (rI+G)^{-1}(I-G)$ and $A_0 = \frac{R}{r}(rI+G)^{-1}G\bar{D}$, thus $A_1 = -\frac{R}{r}(rI+G)^{-1}H$. This establishes Eq. (31).

In the limit $\theta\Sigma_\delta \to \infty$, Eqs. (45) and (46) imply that A_1 diverges while A_2 approaches a finite quantity. In turn, this means that F^{-1} approaches 0. The consequent limiting behavior of Eqs. (44), (45), and (46) can be summarized as

$$\begin{aligned} -RA_0 + \left(I - HC_{2D}^{-1}\right)C_1 + G\bar{D} + HC_{2D}^{-1}C_1 &= 0, \\ -RA_1 - H - \theta\Sigma_\delta\left[\Sigma_\delta^{-1}C_{2D}\Sigma_\eta C_{2D} + I\right] &= 0, \qquad (54) \\ -RA_2 + I - G &= 0. \end{aligned}$$

Solving for the coefficients (to leading order) gives Eq. (32).

■

Proposition 7: *Assume that all matrices are diagonal. Then in the limits* $H, \theta\Sigma_\delta \to 0$ *and* $\theta\Sigma_\delta \to \infty$, $\partial p_{k,t}/\partial d_{k,t} > 0$ and $\partial p_{k,t}/\partial n_{k,t} < 0$. *Moreover,* $\lim_{H,\theta\Sigma_\delta \to 0} \frac{\partial p_{k,t}}{\partial d_{k,t}} < \lim_{\theta\Sigma_\delta \to \infty} \frac{\partial p_{k,t}}{\partial d_{k,t}}$ *and* $\lim_{H,\theta\Sigma_\delta \to 0} \left|\frac{\partial p_{k,t}}{\partial n_{k,t}}\right| < \lim_{\theta\Sigma_\delta \to \infty} \left|\frac{\partial p_{k,t}}{\partial n_{k,t}}\right|$.

Proof. This follows directly from Proposition 6.

Table 1: Portfolios sorted on profitability. This table shows the characteristics of decile portfolios sorted on profitability. The profitability measure, *PROD*, is the ratio of Compustat annual item "Operating Income Before Depreciation" to "Property Plant and Equipment - Total (Gross)." *SIZE* is the average market capitalization of member firms in millions of dollars. *BM* is the average book-to-market ratio, constructed as in Fama and French (1993). *N* is the average number of firms. *EXRET* is the excess value-weighted return with the t-statistic in parentheses. *ALPHA* is the intercept from the time-series regression of the excess portfolio return on the excess market return and the size, value, and momentum factors, with the t-statistic in parentheses. *, **, and *** represent significance at 10, 5, and 1%, respectively. Based on the firm characteristics at the end of fiscal year $t-1$, we form portfolios in June of calendar year t and measure value-weighted monthly returns from July through next June. We use only ordinary common shares on NYSE, AMEX, and NASDAQ of firms in non-financial industries. Only NYSE firms are used to compute breakpoints for ranking. The final sample runs from July 1968 through December 2006.

PROD rank	*PROD*	*SIZE*	*BM*	*N*	*EXRET*		(t-stat)	*ALPHA*		(t-stat)
1	-1.104	195	1.16	784	0.0013		(0.44)	-0.0025		(-1.56)
2	0.099	957	1.27	221	0.0048	**	(2.42)	-0.0017	*	(-1.75)
3	0.130	1,113	1.18	237	0.0054	***	(2.70)	-0.0005		(-0.54)
4	0.164	1,263	1.11	263	0.0050	**	(2.30)	-0.0012		(-1.32)
5	0.205	1,090	1.00	280	0.0059	**	(2.55)	0.0001		(0.05)
6	0.254	1,099	0.93	273	0.0054	**	(2.41)	0.0004		(0.52)
7	0.311	1,376	0.84	282	0.0031		(1.34)	-0.0003		(-0.39)
8	0.388	1,770	0.77	290	0.0050	**	(2.22)	0.0019	**	(2.31)
9	0.522	1,623	0.69	330	0.0056	**	(2.42)	0.0034	***	(4.09)
10	1.877	1,189	0.61	511	0.0049	*	(1.74)	0.0031	***	(3.43)
10-1					0.0036	*	(1.91)	0.0056	***	(3.02)

Table 2: Portfolios sorted on the book-to-market ratio and profitability. This table shows the characteristics of 25 portfolios formed as the cross section of the book-to-market ratio and profitability quintiles. The panels report the following quantities: Panel A: Profitability, the ratio of Compustat annual item "Operating Income Before Depreciation" to "Property Plant and Equipment - Total (Gross)"; Panel B: Size, the average market capitalization in millions of dollars; Panel C: The book-to-market ratio, as described in Fama and French (1993); Panel D: The average number of firms; Panel E: The excess value-weighted return; Panel F: The four-factor alpha, computed as the intercept from the time-series regression of the excess portfolio return on the excess market return and the size, value, and momentum factors. *, **, and *** represent significance at 10, 5, and 1%, respectively. Based on the firm characteristics at the end of fiscal year t – 1, we form portfolios in June of calendar year t and measure value-weighted monthly returns from July through next June. We use only ordinary common shares on NYSE, AMEX, and NASDAQ of firms in non-financial industries. Only NYSE firms are used to compute breakpoints for ranking. The final sample runs from July 1968 through December 2006.

Panel A: Productivity

		BM				
		1	2	3	4	5
	1	-1.89	-0.96	-0.44	-0.33	-0.27
	2	0.15	0.15	0.15	0.15	0.14
PROD	3	0.23	0.23	0.23	0.23	0.22
	4	0.36	0.35	0.35	0.34	0.34
	5	1.26	1.17	1.31	2.03	1.26

Panel B: Size ($ million)

		BM				
		1	2	3	4	5
	1	190	374	477	531	284
	2	1,630	2,408	1,605	990	452
PROD	3	2,630	1,465	861	566	367
	4	3,774	1,426	682	395	355
	5	2,600	790	394	343	326

Panel C: Book-to-market ratio

		BM				
		1	2	3	4	5
	1	0.22	0.55	0.79	1.07	2.20
	2	0.29	0.56	0.79	1.07	1.93
PROD	3	0.29	0.55	0.79	1.06	1.86
	4	0.29	0.55	0.78	1.06	1.86
	5	0.26	0.55	0.78	1.06	1.87

Panel D: Number of stocks

		BM 1	2	3	4	5
	1	276	124	124	164	318
	2	55	67	93	123	162
PROD	3	89	113	114	113	124
	4	146	142	116	89	79
	5	362	190	122	90	76

Panel E: Excess returns

		BM 1	2	3	4	5	5-1
	1	-0.0033	0.0024	0.0028	0.0040 **	0.0072 ***	0.0105 ***
	2	0.0024	0.0050 **	0.0055 **	0.0071 ***	0.0089 ***	0.0065 **
PROD	3	0.0023	0.0066 ***	0.0069 ***	0.0087 ***	0.0093 ***	0.0070 ***
	4	0.0037	0.0043 *	0.0076 ***	0.0083 ***	0.0090 ***	0.0053 **
	5	0.0048 *	0.0080 ***	0.0080 ***	0.0068 **	0.0082 ***	0.0034
	5-1	0.0081 ***	0.0056 ***	0.0052 ***	0.0028	0.0010	

Panel F: Four-factor alphas

		BM 1	2	3	4	5	5-1
	1	-0.0056 **	-0.0035 *	-0.0029 *	-0.0021 **	-0.0011	0.0044 *
	2	-0.0025	-0.0010	-0.0005	-0.0005	-0.0002	0.0023
PROD	3	-0.0013	0.0007	0.0014	0.0007	0.0028	0.0041 *
	4	0.0013	0.0004	0.0012	0.0015	0.0011	-0.0002
	5	0.0036 ***	0.0026 **	0.0017	-0.0002	-0.0003	-0.0039 **
	5-1	0.0092 ***	0.0061 ***	0.0046 **	0.0019	0.0008	

Table 3: Portfolios sorted on size, the book-to-market ratio, and profitability. This table shows the characteristics of 27 portfolios formed as the cross section of the size, book-to-market ratio, and profitability terciles. Profitability is measured by the ratio of Compustat annual item "Operating Income Before Depreciation" to "Property Plant and Equipment - Total (Gross)." The panels report the following quantities: Panel A: Size, the average market capitalization in millions of dollars; Panel B: The book-to-market ratio, as described in Fama and French (1993); Panel C: The value-weighted return on a zero-investment portfolio that goes long highest profitability firms and short lowest profitability firms; Penal D: The four-factor alpha, computed as the intercept from the time-series regression of the zero-investment portfolio return on the excess market return and the size, value, and momentum factors. *, **, and *** represent significance at 10, 5, and 1%, respectively. Based on the firm characteristics at the end of fiscal year $t - 1$, we form portfolios in June of calendar year t and measure value-weighted monthly returns from July through next June. We use only ordinary common shares on NYSE, AMEX, and NASDAQ of firms in non-financial industries. Only NYSE firms are used to compute breakpoints for ranking. The final sample runs from July 1968 through December 2006.

Panel A: Size ($ million)

(i) Low productivity portfolios

		SIZE		
		1	2	3
	1	57	515	5,062
BM	2	59	608	5,356
	3	44	623	3,728

(ii) High productivity portfolios

		SIZE		
		1	2	3
	1	96	604	9,558
BM	2	80	561	4,421
	3	54	520	7,216

Panel B: Book-to-market ratio

(i) Low productivity portfolios

		SIZE		
		1	2	3
	1	0.28	0.32	0.39
BM	2	0.80	0.83	0.83
	3	2.04	1.55	1.43

(ii) High productivity portfolios

		SIZE		
		1	2	3
	1	0.35	0.32	0.29
BM	2	0.78	0.73	0.71
	3	1.63	1.41	1.37

Panel C: Zero-investment portfolio returns (high - low productivity)

		SIZE		
		1	2	3
	1	0.0072 ***	0.0076 ***	0.0034
BM	2	0.0017	0.0030 **	0.0024
	3	-0.0003	-0.0002	0.0001

Panel D: Four-factor alphas on zero-investment portfolios (high - low productivity)

		SIZE		
		1	2	3
	1	0.0068 ***	0.0061 ***	0.0049 **
BM	2	0.0012	0.0024	0.0019
	3	-0.0005	0.0009	0.0017